Level 2 Mathematics and Statistics: practice paper 2026

AS 91262 Apply calculus methods in solving problems · Vertex Academy practice paper, not an NZQA paper

Attempt ALL the questions. Show ALL working. Use NZQA's Level 2 formulae sheet. Write your answers on lined paper, or in the space under each question.

QUESTION ONE

(a) A function is given by $f(x) = x^4 - 3x^2 + 5x - 2$. Use calculus to find the gradient of the function at the point where $x = -2$.

(b) The curve $y = \left(\frac{1}{3}\right)x^3 - 2x^2 + 3x + 1$ is drawn on a graph.

(i) Find the equation of the tangent to the curve at the point where $x = 4$.

(ii) Find the coordinates of the other point on the curve where the gradient is the same as at $x = 4$.

(c) The gradient function of a curve is $f'(x) = 6x^2 - 6x - 12$. The local minimum of the curve has a y-value of -13.
Find the equation of the curve, $f(x)$, and the coordinates of its local maximum.

(d) A tram in Christchurch starts from rest at a stop. For the first 20 seconds its acceleration is $a = 2 - 0.1t$ m s^{-2}, where t is the time in seconds after it starts. At the moment the tram starts, a cyclist riding at a steady 6 m s^{-1} passes the tram, going the same way.
(i) Use calculus to find the tram's velocity and its distance from the stop after t seconds.
(ii) Find how far from the stop the tram catches up with the cyclist.
(iii) Find the greatest distance the cyclist is ahead of the tram before the tram catches up, and use calculus to show it is a maximum.

(e) For a school gala, a stall's archway is shaped like the curve $y = 4 - x^2$, above the x-axis, with x and y in metres. A rectangular banner is hung inside the arch. Its bottom edge sits on the x-axis and its top two corners touch the arch, at $(x, 4 - x^2)$ and $(-x, 4 - x^2)$.

(i) Show that the area of the banner is $A = 8x - 2x^3$.

(ii) Use calculus to find the largest possible banner area, and prove that it is a maximum.

(iii) Another archway has the shape $y = k^2 - x^2$, where k is a positive constant. Show that the largest banner that fits has an area of $(\frac{4\sqrt{3}}{9})k^3$ m², and that its height is always two-thirds of the arch's height.

QUESTION TWO

(a) Water flows into a tank on a Northland farm. The volume of water in the tank t minutes after the tap is turned on is $V = 200 + 30t - 0.5t^2$ litres, for $0 \le t \le 30$.
Use calculus to find the rate at which the volume is changing 10 minutes after the tap is turned on.

(b) The graph of $y = f(x)$ is a cubic. Reading from left to right, it rises to a local maximum at $(-1, 5)$, falls to a local minimum at $(3, -27)$, then rises again for all larger x.
(i) Describe the graph of the gradient function $y = f'(x)$. Give its shape, where it crosses the x-axis, the values of x for which it is below the x-axis, and the x-value of its turning point.
(ii) The cubic is $f(x) = x^3 - 3x^2 - 9x$. Use calculus to confirm your description in (i).

(c) Find the coordinates of the turning points of the curve $y = -x^3 + 6x^2 - 9x + 2$, and use calculus to justify the nature of each one.

(d) A function is given by $f(x) = x^3 + ax^2 + bx + 4$, where a and b are constants. The graph of f has a stationary point at $(2, -8)$.

(i) Use calculus to find the values of a and b.

(ii) Find the coordinates of the other stationary point.

(iii) Use calculus to determine the nature of both stationary points.

(e) This question is about tangents to the curve $y = x^3$.

(i) Find the equation of the tangent to $y = x^3$ at the point where $x = 2$, and find where this tangent meets the curve again.

(ii) Prove that the tangent at the point where $x = p$ ($p \neq 0$) always meets the curve again at the point where $x = -2p$.

(iii) Find the equations of the two tangents to $y = x^3$ that pass through the point $(2, 0)$.

QUESTION THREE

(a) A remote-control boat moves in a straight line on a pond. Its displacement from the jetty after t seconds is $s = 2t^3 - 15t^2 + 36t$ metres.

Find the velocity of the boat at $t = 1$ and its acceleration at $t = 4$.

(b) A stone is thrown straight up from the top of a cliff at Muriwai, 20 m above the sea, with a speed of 15 m s^{-1}. Its acceleration is -9.8 m s^{-2} (taking upwards as positive).

Use calculus to find the greatest height of the stone above the sea, and the time it takes to hit the water.

(c) A camping tent is shaped like a pyramid with a square base. As tents of different sizes are made, the height is always half of the base side length. The volume of a pyramid is $(\frac{1}{3})$ × base area × height. Find the rate of change of the tent's volume with respect to its base side length when the volume is 36 m^3.

(d) A function is given by $f(x) = x^3 - 3kx^2 + 4$, where k is a positive constant.
(i) Use calculus to find, in terms of k, the values of x for which f(x) is decreasing.
(ii) Find the value of k for which the local minimum of f(x) lies on the x-axis, and justify that it is a minimum.

(e) A cylindrical tin is placed inside a cone so that it stands on the cone's base and its top rim touches the sloping side all the way round. The cone has a base radius of 30 cm and a height of 60 cm. The tin has radius r cm and height h cm. The volume of a cylinder is $\pi r^2 h$ and the volume of a cone is $(\frac{1}{3}) \times$ base area $\times$ height.

(i) Use similar triangles to show that $h = 60 - 2r$.

(ii) Use calculus to find the largest possible volume of the tin, and prove that it is a maximum.

(iii) Prove that, for any cone of base radius R and height H, the largest such tin takes up exactly $\frac{4}{9}$ of the cone's volume.

© Vertex Academy 2026. Free to copy for study. vertexacademy.co.nz/ncea-level-2-calculus-practice-paper