Assessment schedule: practice paper 2026

AS 91262 Apply calculus methods in solving problems · Vertex Academy practice paper, not an NZQA paper

QUESTION ONE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f'(x) = 4x^3 - 6x + 5$ $f'(-2) = 4(-8) - 6(-2) + 5$ $f'(-2) = -32 + 12 + 5$ Gradient = −15	Correct derivative AND correct value $f'(-2) = -15$.		
(b)	$\frac{dy}{dx} = x^2 - 4x + 3$ at x = 4: m = 3, $y = \frac{7}{3}$ $y - \frac{7}{3} = 3(x - 4)$ $y = 3x - \frac{29}{3}$ $x^2 - 4x + 3 = 3$, so $x(x - 4) = 0$ (0, 1)	Correct derivative AND correct gradient at x = 4.	Correct equation of the tangent AND the point (0, 1) found.	
(c)	$f(x) = 2x^3 - 3x^2 - 12x + c$ $6(x - 2)(x + 1) = 0$, so x = 2 or −1 $f''(x) = 12x - 6$: $f''(2) = 18 > 0$ min, $f''(-1) = -18 < 0$ max $f(2) = -20 + c = -13$, so c = 7 $f(x) = 2x^3 - 3x^2 - 12x + 7$ Local maximum (−1, 14)	Correct anti-differentiation including + c, or x = 2 and x = −1 found.	Full function $f(x) = 2x^3 - 3x^2 - 12x + 7$ found AND the local maximum (−1, 14).	
(d)	$v = 2t - 0.05t^2$ $s = t^2 - \frac{t^3}{60}$ $t^2 - \frac{t^3}{60} = 6t$, so $t^2 - 60t + 360 = 0$ $t = 30 - \sqrt{540} = 6.76$ s $6 \times 6.76 = 40.57$ m from the stop $G = 6t - t^2 + \frac{t^3}{60}$, $\frac{dG}{dt} = 6 - 2t + \frac{t^2}{20} = 0$ $t = 20 - \sqrt{280} = 3.27$ s, G = 9.51 m $\frac{d^2G}{dt^2} = -2 + \frac{t}{10} = -1.67 < 0$, so maximum Greatest gap 9.51 m	Velocity or distance of the tram found correctly, including working for the constant.	Both v and s for the tram found AND the equation $t^2 - t^3/60 = 6t$ formed.	Catch-up point 40.57 m found with the time 6.76 s justified, AND the greatest gap 9.51 m found and shown to be a maximum.
(e)	$A = 2x(4 - x^2) = 8x - 2x^3$ $A' = 8 - 6x^2 = 0$, so $x = \frac{2}{\sqrt{3}} = 1.155$ $A = 2.31 \times 2.67 = 6.16$ m² $A'' = -12x = -13.9 < 0$, so maximum $A = 2k^2x - 2x^3$, $A' = 2k^2 - 6x^2 = 0$, so $x = \frac{k}{\sqrt{3}}$ height $= k^2 - \frac{k^2}{3} = \frac{2k^2}{3}$ $A = 2(\frac{k}{\sqrt{3}})(\frac{2k^2}{3}) = (\frac{4\sqrt{3}}{9})k^3$	Area expression shown AND A′ correct.	x = 1.155 found from A′ = 0 AND the area 6.16 m² found.	Maximum proved with the second derivative or a gradient test, AND the general area $(4\sqrt{3}/9)k^3$ and two-thirds height shown.

QUESTION TWO

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{dV}{dt} = 30 - t$ at t = 10: 30 − 10 = 20 20 litres per minute	Correct derivative AND rate of 20 litres per minute.		
(b)	f′ is a quadratic f′(x) = 0 at x = −1 and x = 3 f′ > 0 for x < −1 and x > 3, f′ < 0 for −1 < x < 3 f′(x) = 3x² − 6x − 9 = 3(x + 1)(x − 3) vertex at x = 1, f′(1) = −12 U-shaped parabola, roots −1 and 3, below the axis for −1 < x < 3, minimum at (1, −12)	Two of: parabola shape, positive (U-shaped), roots at −1 and 3.	All correct: U-shaped parabola, roots at −1 and 3, negative between them, turning point at x = 1, confirmed from f′(x) = 3x² − 6x − 9.	
(c)	$\frac{dy}{dx} = -3x^2 + 12x - 9 = -3(x - 1)(x - 3) = 0$ x = 1, y = −2 and x = 3, y = 2 $\frac{d^2y}{dx^2} = -6x + 12$ at x = 1: 6 > 0, minimum; at x = 3: −6 < 0, maximum (1, −2) local minimum, (3, 2) local maximum	Correct derivative set to zero and x = 1, x = 3 found.	Both turning points found AND the nature of each justified with calculus.	
(d)	f(2) = −8: 2a + b = −10 f′(2) = 0: 4a + b = −12 a = −1, b = −8 f′(x) = 3x² − 2x − 8 = (3x + 4)(x − 2) $x = -\frac{4}{3}$, $f(-\frac{4}{3}) = \frac{284}{27} = 10.52$ f″(x) = 6x − 2: $f''(-\frac{4}{3}) = -10 < 0$ max; f″(2) = 10 > 0 min $(-\frac{4}{3}, \frac{284}{27})$ local maximum, (2, −8) local minimum	One correct equation in a and b, from f(2) = −8 or f′(2) = 0.	a = −1 and b = −8 found AND x = −4/3 found as the other stationary point.	Other stationary point (−4/3, 284/27) found AND the nature of both points justified with calculus.
(e)	at x = 2: m = 12, point (2, 8), so y = 12x − 16 x³ − 12x + 16 = (x − 2)²(x + 4) meets again at (−4, −64) tangent at p: y = 3p²x − 2p³ x³ − 3p²x + 2p³ = (x − p)²(x + 2p), so other point x = −2p through (2, 0): 6p² − 2p³ = 0, so p = 0 or p = 3 y = 0 and y = 27x − 54	Correct tangent y = 12x − 16 at x = 2.	Point (−4, −64) found AND the general tangent y = 3p²x − 2p³ formed.	General result proved by factorising (or equivalent) AND both tangents through (2, 0) found.

QUESTION THREE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)

(a)	$v = \frac{ds}{dt} = 6t^2 - 30t + 36$ $v(1) = 12$ m s^{-1} $a = \frac{dv}{dt} = 12t - 30$ $a(4) = 18$ m s^{-2}	Correct velocity at t = 1 AND correct acceleration at t = 4.		
(b)	$v = 15 - 9.8t$ $h = 20 + 15t - 4.9t^2$ $v = 0$ at $t = 1.531$ s $h(1.531) = 31.48$ m $4.9t^2 - 15t - 20 = 0$, so $t = 4.07$ s 31.48 m, 4.07 s	$v = 15 - 9.8t$ or $h = 20 + 15t - 4.9t^2$ found, with working for the constant.	Greatest height 31.48 m AND the time 4.07 s found.	
(c)	$V = (\frac{1}{3})x^2(\frac{x}{2}) = \frac{x^3}{6}$ $\frac{x^3}{6} = 36$, so $x = 6$ $\frac{dV}{dx} = \frac{x^2}{2}$ $\frac{dV}{dx} = 18$ m^3 per m	$V = x^3/6$ formed AND dV/dx found.	dV/dx = 18 found at x = 6.	
(d)	$f'(x) = 3x^2 - 6kx = 3x(x - 2k)$ $f'(x) = 0$ at $x = 0$ and $x = 2k$ positive parabola, so $f'(x) < 0$ for $0 < x < 2k$ $f''(x) = 6x - 6k$; $f''(2k) = 6k > 0$, so minimum at $x = 2k$ $f(2k) = 4 - 4k^3 = 0$ $k = 1$	Correct derivative $f'(x) = 3x^2 - 6kx$.	$f'(x)$ set to zero and x = 0, x = 2k found.	Decreasing for $0 < x < 2k$ with a suitable argument (shape or second derivative), AND k = 1 found with the minimum justified.
(e)	$\frac{60 - h}{r} = \frac{60}{30}$, so $h = 60 - 2r$ $V = \pi(60r^2 - 2r^3)$, $V' = 6\pi r(20 - r) = 0$, so $r = 20$ $h = 20$, $V = 8000\pi = 25\,133$ cm^3 $V'' = \pi(120 - 12r) = -120\pi < 0$, so maximum general: $h = H(1 - \frac{r}{R})$, $V = \pi H(r^2 - \frac{r^3}{R})$ $V' = 0$ at $r = \frac{2R}{3}$, $V = (\frac{4}{27})\pi R^2 H$ $(\frac{4}{27})\pi R^2 H \div (\frac{1}{3})\pi R^2 H = \frac{4}{9}$	$h = 60 - 2r$ shown, or $V = \pi(60r^2 - 2r^3)$ differentiated correctly.	V' set to zero and r = 20 found, with the volume 8000π cm^3.	Maximum proved with calculus AND the general 4/9 result proved.

Judgement for each question

N0	N1	N2	A3	A4	M5	M6	E7	E8
No appropriate evidence.	Evidence of correct calculus.	1u	2u	3u	1r	2r	1t	2t

Grade score out of 24 (each question's grade: N0 = 0 to E8 = 8): Not Achieved 0 to 7, Achieved 8 to 13, Merit 14 to 19, Excellence 20 to 24 (the 2025 exam's cut scores).