Level 3 Calculus: practice paper 2026

AS 91577 Apply the algebra of complex numbers in solving problems · Vertex Academy practice paper, not an NZQA paper

Attempt ALL the questions. Show ALL working. Use NZQA's Level 3 formulae sheet. Write your answers on lined paper, or in the space under each question.

QUESTION ONE

(a) When the polynomial $2x^3 + kx^2 - 5x + 4$ is divided by $x + 2$, the remainder is -6.
Find the value of k.

(b) Solve the equation $x^2 - 2kx - 2k^2 = 0$ for x, where k is a positive real constant.
Give your answer in terms of k, simplified as far as possible.

(c) Find the values of the real constant m for which the equation $x^2 + (m - 1)x + m + 2 = 0$ has non-real roots.

(d) Solve the equation $z^4 = 8k^4(-1 + \sqrt{3}\,i)$, where k is a positive real constant.
Give your solutions in polar form r cis θ, with $-\pi < \theta \leq \pi$, in terms of k, and describe where they lie on an Argand diagram.

(e) The locus of a complex number z is described by $|z - 4| = 2|z + 2i|$.

(i) Show that the locus is a circle, and find its centre and its exact radius.

(ii) Find the greatest possible value of $|z|$ for points on the locus, and the complex number z where it occurs.

QUESTION TWO

(a) On an Argand diagram, the complex number u is at the point (3, −2) and the complex number w is at the point (−1, 4).
Find uw in the form a + bi, and state which quadrant of the Argand diagram it lies in.

(b) The complex number $w = -2\sqrt{3} + 2i$.
Write w in polar form, and use De Moivre's theorem to find w^6 in the form a + bi.

(c) Find the value of the real constant p for which $\frac{p + 4i}{2 - i}$ is purely imaginary.

State the value of $\frac{p + 4i}{2 - i}$ for this p.

(d) (i) Find the two square roots of $-15 + 8i$, in the form $a + bi$, by letting $(a + bi)^2 = -15 + 8i$, where a and b are real.

(ii) Hence solve $z^2 + z + 4 - 2i = 0$.

(e) The complex number z has $|z| = 1$ and $z \neq 1$. Let $w = \dfrac{1 + z}{1 - z}$.

(i) Find w when $z = \text{cis}(\frac{\pi}{3})$, giving your answer in the form $a + bi$.

(ii) Prove that w is purely imaginary (or zero) for every such z.

QUESTION THREE

(a) Write $\frac{3 - \sqrt{2}}{2\sqrt{2} + 1}$ in the form $a + b\sqrt{2}$, where a and b are integers.

(b) Solve the equation $2z^2 - 6z + 5 = 0$, giving your solutions in the form $a + bi$.

(c) Solve the equation $\sqrt{(2x - 2)} = 1 + \sqrt{x}$.

(d) One solution of the equation $2z^3 + pz^2 + 30z - 13 = 0$ is $z = 2 + 3i$, where p is a real constant. Find the value of p and the other two solutions of the equation.

(e) (i) Find all complex numbers z that satisfy $z^2 = 2i\bar{z}$, where $\bar{z}$ is the conjugate of z. Give your answers in the form $a + bi$.
(ii) Show, without using your answers to (i), that every non-zero solution has $|z| = 2$ and $z^3 = 8i$.

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