Assessment schedule: practice paper 2026

AS 91577 Apply the algebra of complex numbers in solving problems · Vertex Academy practice paper, not an NZQA paper

QUESTION ONE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f(-2) = -6$ $2(-8) + 4k + 10 + 4 = -6$ $4k - 2 = -6$ $k = -1$	Correct value $k = -1$.		
(b)	$x = \frac{2k \pm \sqrt{(4k^2 + 8k^2)}}{2}$ $\sqrt{(12k^2)} = 2\sqrt{3}\,k$ $x = k \pm \sqrt{3}\,k$ $x = k(1 + \sqrt{3})$ or $x = k(1 - \sqrt{3})$	Both solutions in fully simplified form.		
(c)	$b^2 - 4ac < 0$ $(m - 1)^2 - 4(m + 2) < 0$ $m^2 - 6m - 7 < 0$ $(m - 7)(m + 1) < 0$ $-1 < m < 7$	Correct discriminant expression $m^2 - 6m - 7$ (or the inequation) formed.	$-1 < m < 7$ found, with the inequality the right way round.	
(d)	$8k^4(-1 + \sqrt{3}\,i) = 16k^4\,\text{cis}(\frac{2\pi}{3})$ $r^4\,\text{cis}\,4\theta = 16k^4\,\text{cis}(\frac{2\pi}{3} + 2n\pi)$ $r = 2k$, $\theta = \frac{\pi}{6} + \frac{n\pi}{2}$ $\theta = \frac{\pi}{6}, \frac{2\pi}{3}, -\frac{5\pi}{6}, -\frac{\pi}{3}$ $z = 2k\,\text{cis}(\frac{\pi}{6}), 2k\,\text{cis}(\frac{2\pi}{3}), 2k\,\text{cis}(-\frac{\pi}{3}), 2k\,\text{cis}(-\frac{5\pi}{6})$; corners of a square, radius $2k$	Right-hand side written as $16k^4\,\text{cis}(2\pi/3)$, or one correct root.	All four roots correct in polar form with arguments in range.	
(e)	$\sqrt{((x - 4)^2 + y^2)} = 2\sqrt{(x^2 + (y + 2)^2)}$ $(x - 4)^2 + y^2 = 4x^2 + 4(y + 2)^2$ $3x^2 + 3y^2 + 8x + 16y = 0$ $(x + \frac{4}{3})^2 + (y + \frac{8}{3})^2 = \frac{80}{9}$ centre $-\frac{4}{3} - (\frac{8}{3})i$, radius $\frac{4\sqrt{5}}{3}$ (0, 0) satisfies the equation greatest $\lvert z\rvert$ = diameter = $\frac{8\sqrt{5}}{3}$ at $z = -\frac{8}{3} - (\frac{16}{3})i$	Locus correctly written in terms of x and y with both moduli squared.	Circle equation $3x^2 + 3y^2 + 8x + 16y = 0$ (or equivalent) found.	Centre and radius found AND the greatest $\lvert z\rvert$ of $8\sqrt{5}/3$ at $z = -8/3 - (16/3)i$, with clear justification that the origin lies on the circle.

QUESTION TWO

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)

(a)	$u = 3 - 2i$, $w = -1 + 4i$ $(3 - 2i)(-1 + 4i) = -3 + 12i + 2i - 8i^2$ $= -3 + 14i + 8$ $uw = 5 + 14i$, first quadrant	$uw = 5 + 14i$ found AND placed in the first quadrant.		
(b)	$\lvert w \rvert = \sqrt{(12 + 4)} = 4$ $\arg w = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $w^6 = 4^6 \operatorname{cis}(5\pi)$ $w^6 = 4096(-1 + 0i) = -4096$	$w^6 = -4096$ found, using polar form.		
(c)	$\frac{(p + 4i)(2 + i)}{(2 - i)(2 + i)}$ $= \frac{(2p - 4) + (p + 8)i}{5}$ $2p - 4 = 0$ $p = 2$, value $2i$	Expression written as $((2p - 4) + (p + 8)i)/5$, or equivalent with a real denominator.	$p = 2$ AND the value $2i$ found.	
(d)	$a^2 - b^2 = -15$, $2ab = 8$ $a^4 + 15a^2 - 16 = 0$, so $a^2 = 1$ square roots $\pm(1 + 4i)$ $z = \frac{-1 \pm \sqrt{(-15 + 8i)}}{2}$ $z = \frac{-1 \pm (1 + 4i)}{2}$ $z = 2i$ or $z = -1 - 2i$	Both equations $a^2 - b^2 = -15$ and $2ab = 8$ formed, or one square root found.	Both square roots found AND both solutions $z = 2i$ and $z = -1 - 2i$.	
(e)	$z = \frac{1}{2} + (\frac{\sqrt{3}}{2})i$ $w = \frac{3 + \sqrt{3}\,i}{1 - \sqrt{3}\,i} = 4\sqrt{3}\,\frac{i}{4} = \sqrt{3}\,i$ $z\bar{z} = 1$, so $\bar{z} = \frac{1}{z}$ conjugate of $w = \frac{1 + \bar{z}}{1 - \bar{z}} = (1 + \frac{1}{z})/(1 - \frac{1}{z})$ $= \frac{z + 1}{z - 1} = -w$ $a - bi = -a - bi$, so $a = 0$ w is purely imaginary	$w = \sqrt{3}\,i$ found for $z = \operatorname{cis}(\pi/3)$.	Conjugate of w written using $\bar{z} = 1/z$ (or w written in terms of $\cos\theta$ and $\sin\theta$ with a real denominator).	Complete, clearly justified proof that w is purely imaginary for every z with $\lvert z \rvert = 1$, $z \neq 1$.

QUESTION THREE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	multiply by $\frac{2\sqrt{2} - 1}{2\sqrt{2} - 1}$ bottom $= 8 - 1 = 7$ top $= 6\sqrt{2} - 3 - 4 + \sqrt{2} = 7\sqrt{2} - 7$ $\sqrt{2} - 1$	$\sqrt{2} - 1$ found by rationalising the denominator.		
(b)	$z = \frac{6 \pm \sqrt{(36 - 40)}}{4}$ $\sqrt{(-4)} = 2i$ $z = \frac{6 \pm 2i}{4}$ $z = \frac{3}{2} \pm (\frac{1}{2})i$	Both solutions correct in the form $a + bi$.		

(c)	$2x - 2 = 1 + 2\sqrt{x} + x$ $x - 3 = 2\sqrt{x}$ $x^2 - 6x + 9 = 4x$ $(x - 1)(x - 9) = 0$ $x = 9$ works; $x = 1$ gives $0 \neq 2$ $x = 9$	$x - 3 = 2\sqrt{x}$ (or an equation with the roots removed) reached.	$x = 9$ found AND $x = 1$ rejected after checking.	
(d)	$2 - 3i$ is a root $(z - (2 + 3i))(z - (2 - 3i)) = z^2 - 4z + 13$ $(z^2 - 4z + 13)(2z - 1)$ z term: $4 + 26 = 30$ ✓; z^2 term: $-1 - 8 = -9$ $p = -9$; other solutions $z = 2 - 3i$ and $z = \frac{1}{2}$	Conjugate root $2 - 3i$ given AND the factor $z^2 - 4z + 13$ found, OR $p = -9$ found.	$p = -9$ AND both other solutions found by a clear algebraic method.	
(e)	$x^2 - y^2 + 2xyi = 2y + 2xi$ $x^2 - y^2 = 2y$ and $2x(y - 1) = 0$ $x = 0$: $y = 0$ or -2 $y = 1$: $x^2 = 3$ $\lvert z\rvert^2 = 2\lvert z\rvert$, so $\lvert z\rvert = 2$ for $z \neq 0$ $z^3 = 2i\, z\, \bar{z} = 2i\lvert z\rvert^2$ $z = 0, -2i, \sqrt{3} + i, -\sqrt{3} + i$; non-zero solutions have $\lvert z\rvert = 2$ and $z^3 = 8i$	Both equations $x^2 - y^2 = 2y$ and $2xy = 2x$ formed by comparing parts.	At least two non-zero solutions found from one case.	All four solutions found with both cases justified AND $\lvert z\rvert = 2$ and $z^3 = 8i$ shown algebraically for the non-zero solutions.

Judgement for each question

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One partial solution.	1u	2u	3u	1r	2r	1t, but with one minor error.	1t

Grade score out of 24 (each question's grade: N0 = 0 to E8 = 8): Not Achieved 0 to 6, Achieved 7 to 12, Merit 13 to 19, Excellence 20 to 24 (the 2025 exam's cut scores).