Level 3 Calculus: practice paper 2026

AS 91578 Apply differentiation methods in solving problems · Vertex Academy practice paper, not an NZQA paper

Attempt ALL the questions. Show ALL working. Use NZQA's Level 3 formulae sheet. Write your answers on lined paper, or in the space under each question.

QUESTION ONE

(a) Differentiate $f(x) = 3x^2e^{(-4x)}$. You do not need to simplify your answer.

(b) A Ferris wheel at a Christchurch summer fair turns at a steady speed. The height of one seat above the ground, t seconds after the ride starts, is $h = 9 - 8\cos(\frac{\pi t}{30})$ metres.

(i) Find the rate at which the height of the seat is changing when t = 20, and state whether the seat is rising or falling.

(ii) During the first 60 seconds, find the time when the seat is rising fastest, and that greatest rate of climb.

(c) Find the equation of the normal to the curve $y = \frac{x + 1}{x^2 + 3}$ at the point where $x = -1$.

(d) Consider the family of curves $f(x) = x \ln x - ax$, for $x > 0$, where a is a constant.

(i) Show that every curve in the family has exactly one stationary point, at $x = e^{(a - 1)}$, and prove that it is a minimum.

(ii) Prove that, whatever the value of a, the minimum point lies on the line $y = -x$.

(iii) Prove that the tangent to the curve at $x = 1$ passes through the point $(0, -1)$ for every value of a.

(e) A curve is defined by the parametric equations $x = t^2 + 2t$ and $y = t^3 - 12t$.

(i) Find the coordinates of the stationary points of the curve, and use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

(ii) Prove that the curve has no point of inflection, and state the values of t for which the curve is concave down.

QUESTION TWO

(a) Differentiate $y = 4\tan(3x) - \ln(5x^2 + 2)$.

(b) Pāteke (brown teal) are released into a predator-free sanctuary. A model for the number of pāteke in the sanctuary t years after the release is P = 40 + 60te^(−0.25t).
(i) Is the number of pāteke increasing or decreasing when t = 6? Use calculus to justify your answer.
(ii) Find the greatest number of pāteke the model predicts, and use calculus to show that it is a maximum.

(c) The curve $y = x^3 - 6x^2 + 5x$ has two points where the normal is parallel to the line $x - 4y = 8$. Find the coordinates of both points.

(d) Spouting is made from a flat strip of aluminium 30 cm wide. The strip is bent along two lines, 10 cm in from each edge, so the cross-section has a flat base 10 cm wide and two sides 10 cm long. Each side leans outwards at an angle θ to the vertical, where $0 \le \theta < \frac{\pi}{2}$, so the cross-section is a trapezium that is wider at the top.

(i) Show that the area of the cross-section is $A = 100\cos\theta(1 + \sin\theta)$ cm².

(ii) Find the angle θ that gives the greatest cross-sectional area, and that area. Use calculus to prove that your answer gives the greatest area.

(e) A dinghy is pulled towards a wharf by a rope. The rope runs from the bow of the dinghy, at water level, to a winch on the wharf 4 m above the water. The winch pulls the rope in at a steady 0.5 m per second.
(i) How fast is the dinghy moving towards the wharf when it is 3 m (horizontally) from the wharf?
(ii) Prove that the dinghy speeds up as it gets closer to the wharf, even though the rope is pulled in at a steady rate.

QUESTION THREE

(a) A function is defined in pieces:

$f(x) = x^2$ for $x < 1$

$f(x) = 2x - 1$ for $1 \le x < 3$

$f(x) = 11 - 2x$ for $3 \le x < 5$

$f(x) = 4$ for $x \ge 5$

(i) Find the limit of f(x) as x approaches 5 from below, and the limit as x approaches 5 from above.

(ii) For what value(s) of x is f not continuous?

(iii) For what value(s) of x is f not differentiable?

(b) At the Port of Tauranga, grain is poured onto a flat floor at 2 m^3 per minute. It forms a cone whose base radius is always twice its height. How fast is the height of the pile increasing when the pile is 3 m high?

(The volume of a cone is $V = (\frac{1}{3})\pi r^2 h$.)

(c) A curve is defined by the parametric equations $x = 2 + 3\cos t$ and $y = 1 + 2\sin t$. Find the equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(d) After a dose of medicine, the concentration of the medicine in a patient's blood t hours later is modelled by $C = Ate^{-kt}$ mg per litre, where A and k are positive constants.

(i) Show that $\frac{dC}{dt} = Ae^{-kt}(1 - kt)$ and $\frac{d^2C}{dt^2} = Ake^{-kt}(kt - 2)$.

(ii) Prove that the concentration is decreasing fastest when $t = \frac{2}{k}$.

(iii) Prove that the tangent to the graph of C at $t = \frac{2}{k}$ crosses the t-axis at $t = \frac{4}{k}$.

(e) An island is 3 km off a straight coastline. P is the point on the coast nearest the island. A town is on the coast 8 km from P. A power cable will run underwater from the island to a point Q on the coast, x km from P towards the town, then along the coast from Q to the town. Underwater cable costs $5000 per km and cable on land costs $3000 per km.

(i) Show that the cost of the cable, in thousands of dollars, is $C = 5\sqrt{(9 + x^2)} + 3(8 - x)$, for $0 \le x \le 8$.

(ii) Find the value of x that gives the least cost, and that cost. Use calculus to prove that it is the least cost.

(iii) Show that this best value of x would be the same for any town more than $\frac{9}{4}$ km from P.

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