Assessment schedule: practice paper 2026

AS 91578 Apply differentiation methods in solving problems · Vertex Academy practice paper, not an NZQA paper

QUESTION ONE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$u = 3x^2$, $u' = 6x$ $v = e^{(-4x)}$, $v' = -4e^{(-4x)}$ $f'(x) = 6xe^{(-4x)} + 3x^2(-4e^{(-4x)})$ $f'(x) = 6xe^{(-4x)} - 12x^2e^{(-4x)}$	Correct derivative.		
(b)	$\frac{dh}{dt} = 8 \times (\frac{\pi}{30})\sin(\frac{\pi t}{30}) = (\frac{4\pi}{15})\sin(\frac{\pi t}{30})$ $t = 20$: $(\frac{4\pi}{15}) \times \sin(\frac{2\pi}{3}) = (\frac{4\pi}{15})(\frac{\sqrt{3}}{2})$ $\frac{dh}{dt} = \frac{2\pi\sqrt{3}}{15} \approx 0.726$ m per s > 0, so rising greatest when $\sin(\frac{\pi t}{30}) = 1$, $\frac{\pi t}{30} = \frac{\pi}{2}$ $t = 15$ s, rate $\frac{4\pi}{15} \approx 0.838$ m per s	Correct rate of change at $t = 20$ (0.726 m per s), with evidence of the derivative.	Rising stated AND $t = 15$ s with greatest rate 0.838 m per s, with evidence of the derivative.	
(c)	$x = -1$: $y = \frac{0}{4} = 0$ $\frac{dy}{dx} = \frac{(x^2 + 3) - 2x(x + 1)}{(x^2 + 3)^2}$ $= \frac{3 - 2x - x^2}{(x^2 + 3)^2}$ $x = -1$: $\frac{dy}{dx} = \frac{4}{16} = \frac{1}{4}$ $m(\text{normal}) = -4$ $y = -4(x + 1)$ $y = -4x - 4$	Correct derivative.	Correct equation of the normal, with evidence of the derivative.	
(d)	$f'(x) = \ln x + 1 - a$ $\ln x + 1 - a = 0$, so $x = e^{(a - 1)}$ $f''(x) = \frac{1}{x} > 0$ for $x > 0$ $f(e^{(a - 1)}) = e^{(a - 1)}(a - 1) - ae^{(a - 1)} = -e^{(a - 1)}$ minimum $(e^{(a - 1)}, -e^{(a - 1)})$ lies on $y = -x$ $f(1) = -a$, $f'(1) = 1 - a$ $y + a = (1 - a)(x - 1)$, so $y = (1 - a)x - 1$ $x = 0$ gives $y = -1$ for every a	Correct derivative $f'(x) = \ln x + 1 - a$.	$x = e^{(a - 1)}$ found AND shown to be a minimum using $f''(x) = 1/x > 0$ (or a sign test).	Minimum point $(e^{(a - 1)}, -e^{(a - 1)})$ shown to lie on $y = -x$ AND tangent at $x = 1$ shown to pass through $(0, -1)$, with full and clear calculus justification.

(e)	$\frac{dx}{dt} = 2(t + 1)$, $\frac{dy}{dt} = 3(t^2 - 4)$ $\frac{dy}{dx} = \frac{3(t^2 - 4)}{2(t + 1)}$ $\frac{dy}{dt} = 0$: $t = \pm 2$, points $(8, -16)$ and $(0, 16)$ $\frac{d^2y}{dx^2} = [(\frac{3}{2})\frac{t^2 + 2t + 4}{(t + 1)^2]} \div 2(t + 1) = \frac{3(t^2 + 2t + 4)}{4(t + 1)^3}$ $t = 2$: $\frac{1}{3} > 0$, minimum; $t = -2$: $-3 < 0$, maximum $t^2 + 2t + 4 = (t + 1)^2 + 3 \geq 3$, so $\frac{d^2y}{dx^2} \neq 0$ concave down for $t < -1$	Correct expression for dy/dx.	Correct expression for d^2y/dx^2 AND both stationary points found.	Nature of both stationary points correct AND no point of inflection proved ($t^2 + 2t + 4 > 0$) with concave down for $t < -1$.

QUESTION TWO

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{d}{dx[4\tan(3x)]} = 4 \times 3\sec^2(3x) = 12\sec^2(3x)$ $\frac{d}{dx[\ln(5x^2 + 2)]} = \frac{10x}{5x^2 + 2}$ $\frac{dy}{dx} = 12\sec^2(3x) - \frac{10x}{5x^2 + 2}$	Correct derivative.		
(b)	$\frac{dP}{dt} = 60e^{\wedge}(-0.25t) - 15te^{\wedge}(-0.25t)$ $= 60e^{\wedge}(-0.25t)(1 - 0.25t)$ $t = 6$: $\frac{dP}{dt} = -30e^{\wedge}(-1.5) \approx -6.69 < 0$, so decreasing $\frac{dP}{dt} = 0$ when $t = 4$ $\frac{d^2P}{dt^2} = -15e^{\wedge}(-0.25t)(2 - 0.25t) = -15e^{\wedge}(-1) < 0$ at $t = 4$ $P = 40 + \frac{240}{e} \approx 128$ pāteke	Correct decision of decreasing, with correct derivative.	Maximum of 128 pāteke (or 128.3) at $t = 4$ found, AND shown to be a maximum by the second derivative or a sign test.	
(c)	$y = \frac{x}{4} - 2$, so m(normal) $= \frac{1}{4}$ m(tangent) $= -4$ $\frac{dy}{dx} = 3x^2 - 12x + 5 = -4$ $3x^2 - 12x + 9 = 0$, $(x - 1)(x - 3) = 0$ $x = 1$, $y = 0$; $x = 3$, $y = -12$ (1, 0) and (3, −12)	Correct derivative, and tangent gradient of −4 used.	Both coordinates found, with evidence of the derivative.	
(d)	depth $= 10\cos\theta$, extra width each side $= 10\sin\theta$ $A = (\frac{1}{2})(10 + 10 + 20\sin\theta)(10\cos\theta) = 100\cos\theta(1 + \sin\theta)$ $\frac{dA}{d\theta} = 100(\cos^2\theta - \sin\theta - \sin^2\theta)$ $= 100(1 - 2\sin\theta)(1 + \sin\theta)$ $\sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}$ $\frac{d^2A}{d\theta^2} = -100\cos\theta(1 + 4\sin\theta) = -150\sqrt{3} < 0$ $\theta = \frac{\pi}{6}$ (30°), $A = 75\sqrt{3} \approx 129.9$ cm²	Correct expression for dA/dθ.	Area expression shown AND $\theta = \pi/6$ found from $dA/d\theta = 0$.	Greatest area $75\sqrt{3} \approx 129.9$ cm² found, with full and clear calculus justification that it is the maximum.

(e)	$L^2 = x^2 + 16$ $L\frac{dL}{dt} = x\frac{dx}{dt}$ $x = 3$: $L = 5$, $5(-0.5) = 3\frac{dx}{dt}$ $\frac{dx}{dt} = -\frac{5}{6}$, so 0.833 m per s towards the wharf speed $s = \frac{0.5\sqrt{(x^2 + 16)}}{x} = 0.5\sqrt{(1 + \frac{16}{x^2})}$ $\frac{ds}{dx} = -\frac{8}{x^3\sqrt{(1 + \frac{16}{x^2})}} < 0$ for $x > 0$ The dinghy speeds up as it nears the wharf	Correct relationship $L^2 = x^2 + 16$ differentiated with respect to t.	Correct speed of 5/6 ≈ 0.833 m per s, with evidence of related rates.	Speed expressed in terms of x AND shown to increase as x decreases, with full and clear calculus justification.

QUESTION THREE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x \to 5$ from below: $11 - 2(5) = 1$; from above: 4 $x = 1$: 1 and 1; $x = 3$: 5 and 5; $x = 5$: 1 and 4 not continuous at $x = 5$ gradients: $x = 1$: 2 and 2; $x = 3$: 2 and −2 not differentiable at $x = 3$ and $x = 5$	Two out of three correct responses.		
(b)	given $\frac{dV}{dt} = 2$, want $\frac{dh}{dt}$ at $h = 3$ $r = 2h$, so $V = (\frac{1}{3})\pi(2h)^2h = (\frac{4}{3})\pi h^3$ $\frac{dV}{dh} = 4\pi h^2 = 36\pi$ at $h = 3$ $2 = 36\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{1}{18\pi} \approx 0.0177$ m per minute	Correct expression for dV/dh with V written in terms of h only.	Correct value of dh/dt = 0.0177 m per minute, with evidence of dV/dh and the chain rule.	
(c)	$\frac{dx}{dt} = -3\sin t$, $\frac{dy}{dt} = 2\cos t$ $\frac{dy}{dx} = -(\frac{2}{3})\cot t = -\frac{2}{3}$ at $t = \frac{\pi}{4}$ point $(2 + \frac{3\sqrt{2}}{2}, 1 + \sqrt{2})$ $y - (1 + \sqrt{2}) = -(\frac{2}{3})(x - 2 - \frac{3\sqrt{2}}{2})$ $2x + 3y = 7 + 6\sqrt{2}$	Correct expression for dy/dx.	Correct equation of the tangent (accept 2x + 3y = 15.49 or equivalent), with evidence of dy/dx.	
(d)	$\frac{dC}{dt} = Ae^{(-kt)} - Akte^{(-kt)} = Ae^{(-kt)}(1 - kt)$ $\frac{d^2C}{dt^2} = -Ake^{(-kt)}(1 - kt) - Ake^{(-kt)} = Ake^{(-kt)}(kt - 2)$ $\frac{d^2C}{dt^2} = 0$ at $t = \frac{2}{k}$, negative before and positive after $\frac{dC}{dt}$ least at $t = \frac{2}{k}$, value $-\frac{A}{e^2} < 0$ point $(\frac{2}{k}, \frac{2A}{ke^2})$, gradient $-\frac{A}{e^2}$ $0 - \frac{2A}{ke^2} = -(\frac{A}{e^2})(t - \frac{2}{k})$, so $t - \frac{2}{k} = \frac{2}{k}$ $t = \frac{4}{k}$	Correct expression for dC/dt.	Correct expressions for dC/dt and d^2C/dt^2, AND $t = 2/k$ found from $d^2C/dt^2 = 0$.	Decreasing fastest at $t = 2/k$ justified by the sign change of d^2C/dt^2 AND the tangent shown to cross the t-axis at $t = 4/k$, with full and clear calculus justification.

(e)	underwater = $\sqrt{(3^2 + x^2)}$, land = $8 - x$ $C = 5\sqrt{(9 + x^2)} + 3(8 - x)$ $\frac{dC}{dx} = \frac{5x}{\sqrt{(9 + x^2)}} - 3$ $5x = 3\sqrt{(9 + x^2)}$, $25x^2 = 81 + 9x^2$, $x = \frac{9}{4}$ $\frac{d^2C}{dx^2} = \frac{45}{(9 + x^2)^{\wedge}(\frac{3}{2})} > 0$ $C(\frac{9}{4}) = 18.75 + 17.25 = 36$, so \$36 000 town D km away: $\frac{d}{dx[3(D - x)]} = -3$, same $\frac{dC}{dx}$, so x $= \frac{9}{4}$ for $D > \frac{9}{4}$	Correct expression for dC/dx (accept from a model with a minor error).	x = 9/4 found from dC/dx = 0.	Least cost \$36 000 found with full and clear calculus justification of the minimum AND x = 9/4 shown to be independent of the distance to the town.

Judgement for each question

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t: correct solution but with one minor error.	2t: correct solution with full and clear calculus justification.

Grade score out of 24 (each question's grade: N0 = 0 to E8 = 8): Not Achieved 0 to 7, Achieved 8 to 13, Merit 14 to 19, Excellence 20 to 24 (the 2025 exam's cut scores).