Level 3 Calculus: practice paper 2026

AS 91579 Apply integration methods in solving problems · Vertex Academy practice paper, not an NZQA paper

Attempt ALL the questions. Show ALL working. Use NZQA's Level 3 formulae sheet. Write your answers on lined paper, or in the space under each question.

QUESTION ONE

(a) Find $\int \left(\frac{5}{x} + 4\sin(2x) - 6e^{(-3x)}\right) dx$.

(b) Find the exact value of k, where k > 0, such that
$\int$ from 0 to k of $2e^{(2x)}\,dx = 3 \times \int$ from 0 to k of $e^{x}\,dx$.
You must use calculus and show the results of any integration needed to solve the problem.

(c) A jet boat on the Shotover River starts from a jetty and moves along a straight stretch of river. Its acceleration t seconds after leaving is $a(t) = 6t - 12$ m s^{-2}. Its velocity when it leaves the jetty is 9 m s^{-1}. Find the total distance the boat travels in the first 4 seconds, and how far from the jetty it is at $t = 4$. You must use calculus and show the results of any integration needed to solve the problem.

(d) The curves $y = \cos x$ and $y = \sin(2x)$ are drawn for $0 \le x \le \frac{\pi}{2}$. Between $x = 0$ and $x = \frac{\pi}{6}$ the graph of $y = \cos x$ is above $y = \sin(2x)$; between $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ it is below.

(i) Show that the curves meet only at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ in this interval.

(ii) Find the total area enclosed between the two curves from $x = 0$ to $x = \frac{\pi}{2}$.

(iii) Explain why $\int$ from 0 to $\frac{\pi}{2}$ of $(\cos x - \sin(2x))\,dx = 0$, even though the curves enclose a region.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Consider the differential equation $\frac{dy}{dx} = \frac{xy}{x + 2}$, where $y > 0$ and $x \geq 0$.

Given that $y = 2$ when $x = 0$, find the exact value of y when $x = 3$.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

(a) Find $\int \left(\frac{8}{(4x-3)^3} + 3\sec^2(2x)\right) dx$.

(b) A rainwater tank at a Northland marae holds 200 litres when a storm starts. During the storm, water flows into the tank at a rate of R(t) = 40e^(−0.1t) litres per minute, where t is the time in minutes since the storm started. No water leaves the tank.

(i) Find the volume of water in the tank 10 minutes after the storm started.

(ii) The tank holds 650 litres. Show that, if the flow keeps following this model, the tank will never overflow.

You must use calculus and show the results of any integration needed to solve the problem.

(c) The curve $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ enclose a region. Between the points where they meet, the line is above the curve.

Find the area of the enclosed region.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The graph of $y = \sin(3x)\cos x$ is drawn for $0 \leq x \leq \frac{\pi}{2}$. It crosses the x-axis at $x = 0$, $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$.

The curve is above the x-axis between 0 and $\frac{\pi}{3}$, and below it between $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

Find the total area enclosed between the curve and the x-axis from $x = 0$ to $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) A tray of kai from a hāngi is put on a bench to cool. Newton's Law of Cooling says the rate at which the food's temperature T °C falls is proportional to the difference between T and the room temperature A °C.

(i) Write a differential equation for this situation, and solve it to show that $T = A + Be^{-kt}$, where t is in minutes and B and k are constants.

(ii) The room temperature is not known. The food is 80 °C when t = 0, 60 °C when t = 10 and 46 °C when t = 20. Find the room temperature.

(iii) How long after t = 0 does the food reach 20 °C?

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

(a) A surveyor measures the depth of a stream in the Waikato across its 12 m width. The depths, measured every 2 m from one bank, are shown in the table.

Use the trapezium rule to estimate the area of the cross-section of the stream.

Distance from bank (m)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Depth (m)	0.3	0.8	1.4	1.9	1.6	1.1	0.2

(b) Consider the differential equation $\frac{dy}{dx} = \frac{4x + 2}{y}$, where $y > 0$.

Given that $y = 3$ when $x = 1$, find the value of y when $x = 2$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Find the value of the constant a, given that

$\int$ from 1 to 4 of $(a\sqrt{x} - \frac{4}{x})\,dx = 28 - 8\ln 2$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) For a constant $k > 0$, the parabola $y = x^2$ and the line $y = kx$ meet at $x = 0$ and $x = k$. Between these points the line is above the parabola.

(i) Show that the area enclosed by the parabola and the line is $\frac{k^3}{6}$.

(ii) The vertical line $x = p$, where $0 < p < k$, splits the enclosed region into two parts of equal area. Prove that $4p^3 - 6kp^2 + k^3 = 0$.

(iii) Hence show that $p = \frac{k}{2}$ is the only possible value of p.

You must use calculus and show the results of any integration needed to solve the problem.

(e) A water tank at a marae is being drained through a tap at the bottom. The depth of water in the tank is h metres, t minutes after the tap is opened. The depth falls at a rate proportional to the square root of the depth. When the tap is opened the depth is 1.44 m, and 10 minutes later it is 1.00 m.
(i) Form and solve a differential equation to find how long it takes for the tank to empty.
(ii) Show that the tank takes more than twice as long to drain the bottom half of its starting depth as the top half.
You must use calculus and show the results of any integration needed to solve the problem.

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