Assessment schedule: practice paper 2026

AS 91579 Apply integration methods in solving problems · Vertex Academy practice paper, not an NZQA paper

QUESTION ONE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\int \frac{5}{x}\,dx = 5\ln\lvert x\rvert$ $\int 4\sin(2x)\,dx = -2\cos(2x)$ $\int -6e^{(-3x)}\,dx = 2e^{(-3x)}$ $5\ln\lvert x\rvert - 2\cos(2x) + 2e^{(-3x)} + c$	Correct answer.		
(b)	$\int$ from 0 to k of $2e^{(2x)}\,dx = [e^{(2x)}] = e^{(2k)} - 1$ $3\int$ from 0 to k of $e^x\,dx = 3[e^x] = 3e^k - 3$ $u^2 - 3u + 2 = 0$ with $u = e^k$ $u = 1$ ($k = 0$, rejected) or $u = 2$ $k = \ln 2$	Both integrations correct.	$k = \ln 2$ found, with $k = 0$ rejected and evidence of integration.	
(c)	$v = \int (6t - 12)\,dt = 3t^2 - 12t + 9$ $s = \int v\,dt = t^3 - 6t^2 + 9t$ $v = 0$ at $t = 1$ and $t = 3$ $s(1) = 4$, $s(3) = 0$, $s(4) = 4$ distance $= 4 + 4 + 4 = 12$ m 12 m travelled; 4 m from the jetty	Correct velocity formula, including the constant ($v = 3t^2 - 12t + 9$).	Total distance of 12 m found (with the change of direction handled), with evidence of integration.	
(d)	$\cos x = 2\sin x \cos x$, so $\cos x(2\sin x - 1) = 0$ $x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$ $\int$ from 0 to $\frac{\pi}{6}$ of $(\cos x - \sin 2x)\,dx = [\sin x + (\frac{1}{2})\cos 2x] = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ $\int$ from $\frac{\pi}{6}$ to $\frac{\pi}{2}$ of $(\sin 2x - \cos x)\,dx = [-(\frac{1}{2})\cos 2x - \sin x] = -\frac{1}{2} + \frac{3}{4} = \frac{1}{4}$ net integral $= \frac{1}{4} - \frac{1}{4} = 0$ total area $= \frac{1}{2}$ units²	One correct integration, or both intersection points found from $\cos x(2\sin x - 1) = 0$.	Total area of 1/2 units² found, with evidence of integration.	Full solution: intersections shown, area 1/2 found AND a clear explanation that the net integral is 0 because the equal areas have opposite signs.
(e)	$\int (\frac{1}{y})\,dy = \int \frac{x}{x+2}\,dx$ $\frac{x}{x+2} = 1 - \frac{2}{x+2}$ $\ln y = x - 2\ln(x + 2) + c$ $\ln 2 = -2\ln 2 + c$, so $c = 3\ln 2$ $\ln y = 3 - 2\ln 5 + 3\ln 2 = 3 + \ln(\frac{8}{25})$ $y = \frac{8e^3}{25} \approx 6.43$	One of the integrations correct.	Both integrations correct AND the value of c found.	$y = 8e^3/25 \approx 6.43$ found, with full and clear working (E7: correct value of ln y; E8: correct value of y).

QUESTION TWO

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	$8(4x-3)^{(-3)} \rightarrow \frac{8(4x-3)^{(-2)}}{-2\times 4}$ $= -\frac{1}{(4x-3)^2}$ $\int 3\sec^2(2x)\,dx = (\frac{3}{2})\tan(2x)$ $-\frac{1}{(4x-3)^2} + (\frac{3}{2})\tan(2x) + c$	Correct answer.		
(b)	$V = \int 40e^{(-0.1t)}\,dt = -400e^{(-0.1t)} + c$ $200 = -400 + c$, so $c = 600$ $V = 600 - 400e^{(-0.1t)}$ $V(10) = 600 - 400e^{(-1)} \approx 452.85$ $400e^{(-0.1t)} > 0$, so $V < 600 < 650$ for all t (i) about 453 litres (ii) never more than 600 litres, so no overflow	Correct formula for V, including the value of c.	V(10) ≈ 453 litres found AND a valid argument that V stays below 600 litres, with evidence of integration.	
(c)	$x^2 - 5x + 4 = 0$, so $x = 1$, $x = 4$ Area = $\int$ from 1 to 4 of $(-x^2 + 5x - 4)\,dx$ $= [-\frac{x^3}{3} + \frac{5x^2}{2} - 4x]$ from 1 to 4 $= \frac{8}{3} - (-\frac{11}{6}) = \frac{27}{6}$ 4.5 units²	Intersections x = 1 and x = 4 found AND a correct integral of (line − curve).	Area of 4.5 units² found, with evidence of integration.	
(d)	$\sin 3x \cos x = (\frac{1}{2})(\sin 4x + \sin 2x)$ $F(x) = -(\frac{1}{8})\cos 4x - (\frac{1}{4})\cos 2x$ $F(0) = -\frac{3}{8}$, $F(\frac{\pi}{3}) = \frac{3}{16}$, $F(\frac{\pi}{2}) = \frac{1}{8}$ $\int$ from 0 to $\frac{\pi}{3} = \frac{9}{16}$ $\int$ from $\frac{\pi}{3}$ to $\frac{\pi}{2} = -\frac{1}{16}$, area $\frac{1}{16}$ total area $= \frac{5}{8} = 0.625$ units²	Function correctly rewritten as (1/2)(sin 4x + sin 2x), or a correct integral.	Correct integration AND either 9/16 or 1/16 found.	Total area of 5/8 = 0.625 units² found, with the region below the axis handled and full justification.
(e)	$\frac{dT}{dt} = -k(T - A)$ $\int \frac{1}{T-A}\,dT = \int -k\,dt$, so $\ln(T - A) = -kt + c$ $T = A + Be^{(-kt)}$ $(60 - A)^2 = (80 - A)(46 - A)$, so $6A = 80$ $A = \frac{40}{3}$, $B = \frac{200}{3}$, $e^{(-10k)} = 0.7$, $k \approx 0.0357$ $e^{(-kt)} = 0.1$, $t = 10\ln\frac{10}{\ln(\frac{10}{7})}$ (ii) about 13.3 °C (iii) about 64.6 minutes	Correct differential equation and one correct integration.	T = A + Be^(−kt) derived by integration AND an equation formed that leads to the room temperature.	Room temperature 40/3 ≈ 13.3 °C AND time of 64.6 minutes found, with full and clear calculus justification.

QUESTION THREE

Part	Evidence	Achievement (u)	Merit (r)	Excellence (t)

(a)	h = 2 ends: 0.3 + 0.2 = 0.5 middles: 2 × (0.8 + 1.4 + 1.9 + 1.6 + 1.1) = 13.6 area ≈ $(\frac{2}{2})(0.5 + 13.6)$ 14.1 m²	Correct estimate of 14.1 m².		
(b)	$\int y\,dy = \int (4x + 2)\,dx$ $\frac{y^2}{2} = 2x^2 + 2x + c$ $\frac{9}{2} = 4 + c$, so $c = \frac{1}{2}$ $y^2 = 4x^2 + 4x + 1$ x = 2: $y^2 = 25$, y = 5 (y > 0)	Both integrations correct.	y = 5 found, with evidence of integration and the constant evaluated.	
(c)	$\int (a\,x^{(\frac{1}{2})} - \frac{4}{x})\,dx = (\frac{2a}{3})x^{(\frac{3}{2})} - 4\ln x$ [ ] from 1 to 4 = $(\frac{16a}{3} - 8\ln 2) - \frac{2a}{3}$ $\frac{14a}{3} - 8\ln 2 = 28 - 8\ln 2$ a = 6	Correct integration.	a = 6 found, with evidence of integration.	
(d)	$\int$ from 0 to k of $(kx - x^2)\,dx = \frac{k^3}{2} - \frac{k^3}{3} = \frac{k^3}{6}$ $\int$ from 0 to p of $(kx - x^2)\,dx = \frac{kp^2}{2} - \frac{p^3}{3} = \frac{k^3}{12}$ $6kp^2 - 4p^3 = k^3$, so $4p^3 - 6kp^2 + k^3 = 0$ $4p^3 - 6kp^2 + k^3 = (2p - k)(2p^2 - 2kp - k^2)$ other roots $p = \frac{k(1 \pm \sqrt{3})}{2}$, outside $0 < p < k$ $p = \frac{k}{2}$ only	Correct integration of $kx - x^2$.	Area $k^3/6$ shown AND the equation for p formed from half the area.	Cubic proved AND p = k/2 shown to be the only root between 0 and k, with full and clear justification.
(e)	$\frac{dh}{dt} = -k\sqrt{h}$ $\int h^{(-\frac{1}{2})}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + c$ c = 2.4 k = 0.04 h = 0: $t = \frac{2.4}{0.04}$ = 60 minutes h = 0.72: $t = \frac{2.4 - 2\sqrt{0.72}}{0.04} \approx 17.6$; rest ≈ 42.4 > 2 × 17.6 (i) 60 minutes (ii) 42.4 min > 35.2 min	Correct differential equation and integration ($2\sqrt{h} = -kt + c$).	c and k found AND the time to empty of 60 minutes.	Times for both halves found (about 17.6 and 42.4 minutes) AND compared to show the bottom half takes more than twice as long, with full and clear calculus justification.

Judgement for each question

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t: correct solution but with one minor error.	2t: correct solution with full and clear calculus justification.

Grade score out of 24 (each question's grade: N0 = 0 to E8 = 8): Not Achieved 0 to 6, Achieved 7 to 12, Merit 13 to 18, Excellence 19 to 24 (the 2025 exam's cut scores).