NCEA Level 2 · AS 91256: Lines, Points & the Coordinate Plane

Name: _____ Date: _____

This standard is all about working with points and straight lines on the number plane, distances, midpoints, gradients, equations of lines, and parallel/perpendicular relationships. Work through each section in order; check your working against the answers, and note the 'evidence' hints if you're aiming for Merit or Excellence.

Revision summary

- Distance between $A(x_1,y_1)$ and $B(x_2,y_2)$: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$.
- Midpoint of AB: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
- Gradient: $m = \frac{y_2-y_1}{x_2-x_1}$ = rise ÷ run.
- Equation of a line: $y = mx + c$, or $y - y_1 = m(x - x_1)$ given a point and gradient.
- Parallel lines have equal gradients ($m_1 = m_2$).
- Perpendicular lines have gradients that multiply to −1 ($m_1 \times m_2 = -1$), so $m_2 = -1/m_1$.
- To find where two lines meet, solve their equations simultaneously.
- Perpendicular bisector of AB: passes through the midpoint of AB with gradient −1/(gradient of AB).

Key skills

1. Find the distance between A(1, 2) and B(7, 10).

2. Find the midpoint of P(−3, 5) and Q(7, −1).

3. Find the gradient of the line through C(2, 3) and D(6, 11).

4. Write the equation of the line with gradient 3 that passes through (2, −1).

Achieved

5. Find the equation of the straight line passing through the points (1, 2) and (4, 11).

6. A line has equation $y = 2x + 5$. Find the equation of the line perpendicular to it that passes through (4, 1).

7. Find the equation of the line parallel to $y = -3x + 2$ that passes through (2, 5).

8. On a walking-track map (units in km), a café is at (2, 3) and a lookout is at (8, 11). How far apart are they in a straight line?

Merit

9. A triangle has vertices A(−2, 1), B(4, 3) and C(2, 9). Show that the triangle has a right angle, and state where it is.

10. Two straight roads are modelled by $y = 2x - 3$ and $x + y = 9$. Find the coordinates of the point where the roads cross.

11. Find the equation of the perpendicular bisector of the line segment joining A(1, 2) and B(5, 8).

Excellence

12. A quadrilateral has vertices A(1, 1), B(5, 3), C(7, 7) and D(3, 5). Prove that ABCD is a rhombus.

13. A stream through a park follows the line $y = 2x$ (the park entrance is the origin). A pōhutukawa tree stands at T(5, 0). Find the shortest distance from the tree to the stream, and the coordinates of the nearest point on the stream. Justify why this is the shortest distance.

14. Two lines are given by $y = (k - 1)x + 3$ and $2x + 4y = 7$, where k is a constant. Find the value of k that makes the lines perpendicular, and explain your reasoning.

Answer key (for parents and tutors)

1. Find the distance between A(1, 2) and B(7, 10).
   $d = \sqrt{((7-1)^2 + (10-2)^2)} = \sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$.
   Achieved: applies the distance formula.

2. Find the midpoint of P(−3, 5) and Q(7, −1).
   $M = ((-3+7)/2 , (5+(-1))/2) = (4/2 , 4/2) = (2, 2)$.
   Achieved: applies the midpoint formula.

3. Find the gradient of the line through C(2, 3) and D(6, 11).
   $m = (11 - 3)/(6 - 2) = 8/4 = 2$.
   Achieved: applies the gradient formula.

4. Write the equation of the line with gradient 3 that passes through (2, −1).
   $y - (-1) = 3(x - 2) \rightarrow y + 1 = 3x - 6 \rightarrow y = 3x - 7$.
   Achieved: forms a line equation from a point and gradient.

5. Find the equation of the straight line passing through the points (1, 2) and (4, 11).
   Gradient $m = (11 - 2)/(4 - 1) = 9/3 = 3$. Using (1, 2): $y - 2 = 3(x - 1) \rightarrow y = 3x - 3 + 2 \rightarrow y = 3x - 1$.
   Achieved: selects gradient then forms the line equation.

6. A line has equation $y = 2x + 5$. Find the equation of the line perpendicular to it that passes through (4, 1).
   Gradient of given line = 2, so perpendicular gradient = $-1/2$. Using (4, 1): $y - 1 = -1/2(x - 4) \rightarrow y - 1 = -1/2\,x + 2 \rightarrow y = -1/2\,x + 3$.
   Achieved: applies the perpendicular gradient rule.

7. Find the equation of the line parallel to $y = -3x + 2$ that passes through (2, 5).
   Parallel lines have equal gradients, so $m = -3$. Using (2, 5): $y - 5 = -3(x - 2) \rightarrow y - 5 = -3x + 6 \rightarrow y = -3x + 11$.
   Achieved: applies the parallel gradient rule.

8. On a walking-track map (units in km), a café is at (2, 3) and a lookout is at (8, 11). How far apart are they in a straight line?
   $d = \sqrt{((8-2)^2 + (11-3)^2)} = \sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$ km.
   Achieved: applies distance formula in a modelling context.

9. A triangle has vertices A(−2, 1), B(4, 3) and C(2, 9). Show that the triangle has a right angle, and state where it is.
   Gradient $AB = (3 - 1)/(4 - (-2)) = 2/6 = 1/3$. Gradient $BC = (9 - 3)/(2 - 4) = 6/(-2) = -3$. Product of gradients = $(1/3) \times (-3) = -1$, so AB is perpendicular to BC. Therefore the right angle is at B.
   Merit: links gradients of two sides to prove perpendicularity.

10. Two straight roads are modelled by $y = 2x - 3$ and $x + y = 9$. Find the coordinates of the point where the roads cross.
    Substitute $y = 2x - 3$ into $x + y = 9$: $x + (2x - 3) = 9 \rightarrow 3x - 3 = 9 \rightarrow 3x = 12 \rightarrow x = 4$. Then $y = 2(4) - 3 = 5$. The roads cross at (4, 5).
    Merit: connects two representations by solving simultaneously.

11. Find the equation of the perpendicular bisector of the line segment joining A(1, 2) and B(5, 8).
    Midpoint of $AB = ((1+5)/2 , (2+8)/2) = (3, 5)$. Gradient $AB = (8 - 2)/(5 - 1) = 6/4 = 3/2$, so perpendicular gradient = $-2/3$. Through (3, 5): $y - 5 = -2/3(x - 3) \rightarrow y - 5 = -2/3\,x + 2 \rightarrow y = -2/3\,x + 7$.
    Merit: combines midpoint and perpendicular gradient in a multi-step solution.

12. A quadrilateral has vertices A(1, 1), B(5, 3), C(7, 7) and D(3, 5). Prove that ABCD is a rhombus.
    Gradients: $AB = (3-1)/(5-1) = 2/4 = 1/2$; $DC = (7-5)/(7-3) = 2/4 = 1/2$, so $AB \parallel DC$. $AD = (5-1)/(3-1) = 4/2 = 2$; $BC = (7-3)/(7-5) = 4/2 = 2$, so $AD \parallel BC$. Both pairs of opposite sides are parallel, so ABCD is a parallelogram. Side lengths: $AB = \sqrt{(4^2 + 2^2)} = \sqrt{20}$; $AD = \sqrt{(2^2 + 4^2)} = \sqrt{20}$. Adjacent sides are equal, and since a parallelogram with two equal adjacent sides has all four sides equal (AB = DC and AD = BC by the parallelogram property), all sides = $\sqrt{20}$. A parallelogram with all sides equal is a rhombus. ■
    Excellence: constructs a full logical proof combining gradients and lengths.

13. A stream through a park follows the line $y = 2x$ (the park entrance is the origin). A pōhutukawa tree stands at T(5, 0). Find the shortest distance from the tree to the stream, and the coordinates of the nearest point on the stream. Justify

why this is the shortest distance.

The shortest distance from a point to a line is along the perpendicular, because any other path forms the hypotenuse of a right triangle and is therefore longer. Stream gradient = 2, so the perpendicular gradient = −1/2. Line from T(5, 0): $y - 0 = -1/2(x - 5) \rightarrow y = -1/2\,x + 5/2$. Intersect with $y = 2x$: $2x = -1/2\,x + 5/2 \rightarrow 2.5x = 2.5 \rightarrow x = 1$, $y = 2$. Nearest point N = (1, 2). Distance TN = $\sqrt{((5-1)^2 + (0-2)^2)} = \sqrt{(16 + 4)} = \sqrt{20} = 2\sqrt{5}$ (≈ 4.47 units).

Excellence: solves an unfamiliar problem and justifies the perpendicular gives the minimum.

14. Two lines are given by $y = (k - 1)x + 3$ and $2x + 4y = 7$, where k is a constant. Find the value of k that makes the lines perpendicular, and explain your reasoning.

Rearrange the second line: $4y = -2x + 7 \rightarrow y = -1/2\,x + 7/4$, so its gradient is −1/2. For perpendicular lines the gradients multiply to −1: $(k - 1) \times (-1/2) = -1$. Multiply both sides by −2: $k - 1 = 2$, so k = 3. Reasoning: only when the first gradient (k − 1) is the negative reciprocal of −1/2, i.e. 2, are the lines perpendicular; this occurs at k = 3.

Excellence: generalises the perpendicular condition with an algebraic parameter and justifies it.