NCEA Level 2 · AS 91257: Graphs & Graphical Methods

Name: _____ Date: _____

This standard is all about drawing, reading and transforming graphs, lines, parabolas, exponentials and more, and using them to model real situations. Work through the sections in order and check every step against the worked answers.

Revision summary

- Line: $y = mx + c$, where m = gradient = rise/run and c = y-intercept. Parallel lines share m; perpendicular lines have gradients that multiply to −1.
- Parabola: $y = a(x - h)^2 + k$ has vertex (h, k). $y = a(x - p)(x - q)$ has x-intercepts at p and q; axis of symmetry midway between them.
- Transformations of $y = f(x)$: $y = f(x) + k$ shifts up k; $y = f(x - h)$ shifts right h; $y = af(x)$ stretches vertically by factor a; $y = -f(x)$ reflects in the x-axis.
- Exponential: $y = a \cdot b^x$ has a horizontal asymptote ($y = 0$ unless shifted); $b > 1$ grows, $0 < b < 1$ decays; y-intercept is a.
- Intercepts: set $y = 0$ for x-intercepts (roots), set $x = 0$ for the y-intercept.
- Asymptotes: lines a curve approaches but never reaches, key for exponential and reciprocal ($y = 1/x$) graphs.
- To model: pick the graph type that matches the pattern (constant rate → linear, constant multiplier → exponential, single turning point → quadratic), then fit features to the data.
- Solve simultaneously by finding where two graphs intersect, equal y-values.

Key skills

1. State the gradient and y-intercept of the line $y = 3x - 5$, then find the x-intercept.

2. Write the coordinates of the vertex of $y = (x - 4)^2 + 2$ and state whether it is a maximum or minimum.

3. Describe the transformation that takes $y = x^2$ to $y = x^2 - 6$.

4. For the exponential $y = 2^x$, state the y-intercept and the equation of the horizontal asymptote.

Achieved

5. A water tank at a bach near Muriwai starts with 500 L and is filled at a constant 40 L per minute. Write an equation for the volume V after t minutes and find the volume after 12 minutes.

6. Sketch $y = (x + 1)(x - 5)$ by finding its x-intercepts, axis of symmetry and vertex.

7. The population of a rabbit colony is modelled by $P = 30 \times 1.5^t$, where t is in years. Find the starting population and the population after 3 years.

8. A line passes through (0, 4) and (2, 10). Find its equation in the form $y = mx + c$.

Merit

9. A drone is launched from a 5 m platform. Its height (m) after t seconds is $h = -5t^2 + 20t + 5$. Find the maximum height reached and when it hits the ground.

10. The graph of $y = x^2$ is transformed to $y = 2(x - 3)^2 - 4$. Describe the sequence of transformations, then state the coordinates of the new vertex and the y-intercept.

11. Two courier companies charge for a delivery. FastVan charges $8 plus $2 per km; QuickGo charges a flat $20. Write an equation for each, find the distance where the costs are equal, and state which is cheaper for a 10 km trip.

Excellence

12. A parabola has x-intercepts at $x = -2$ and $x = 6$ and passes through $(2, -32)$. Find its equation in the form $y = ax^2 + bx + c$, and justify why the vertex must lie on $x = 2$.

13. An exponential model $y = a \cdot b^x$ passes through $(0, 12)$ and $(2, 3)$. Find a and b, and explain what value of b tells you about the situation and the behaviour of the graph.

14. Prove that the line $y = x + k$ is tangent to the parabola $y = x^2$ only when $k = -1/4$, and state the point of tangency.

Answer key (for parents and tutors)

1. State the gradient and y-intercept of the line $y = 3x - 5$, then find the x-intercept.
   Gradient $m = 3$, y-intercept $c = -5$. For the x-intercept set $y = 0$: $0 = 3x - 5$, so $3x = 5$, $x = 5/3 \approx 1.67$. x-intercept is $(5/3, 0)$.
   Achieved: reads features directly from $y = mx + c$ form.

2. Write the coordinates of the vertex of $y = (x - 4)^2 + 2$ and state whether it is a maximum or minimum.
   The form $y = a(x - h)^2 + k$ gives vertex $(h, k) = (4, 2)$. Since $a = 1 > 0$ the parabola opens upward, so the vertex is a minimum.
   Achieved: identifies vertex and orientation from completed-square form.

3. Describe the transformation that takes $y = x^2$ to $y = x^2 - 6$.
   Adding -6 to the function shifts every point down by 6 units. It is a vertical translation of 6 units downward.
   Key skill: names a vertical translation.

4. For the exponential $y = 2^x$, state the y-intercept and the equation of the horizontal asymptote.
   At $x = 0$, $y = 2^0 = 1$, so the y-intercept is $(0, 1)$. As $x \to -\infty$, $y \to 0$, so the horizontal asymptote is $y = 0$.
   Key skill: reads intercept and asymptote of a basic exponential.

5. A water tank at a bach near Muriwai starts with 500 L and is filled at a constant 40 L per minute. Write an equation for the volume V after t minutes and find the volume after 12 minutes.
   Constant rate means a linear model: $V = 500 + 40t$. After 12 minutes: $V = 500 + 40 \times 12 = 500 + 480 = 980$ L.
   Achieved: selects a linear model and applies it.

6. Sketch $y = (x + 1)(x - 5)$ by finding its x-intercepts, axis of symmetry and vertex.
   x-intercepts where $y = 0$: $x = -1$ and $x = 5$, i.e. points $(-1, 0)$ and $(5, 0)$. Axis of symmetry is midway: $x = (-1 + 5)/2 = 2$. Vertex y-value: $y = (2 + 1)(2 - 5) = 3 \times (-3) = -9$, so vertex $(2, -9)$. Parabola opens upward ($a = 1$) through the two intercepts with minimum at $(2, -9)$.
   Achieved: finds roots, axis and vertex to sketch a parabola.

7. The population of a rabbit colony is modelled by $P = 30 \times 1.5^t$, where t is in years. Find the starting population and the population after 3 years.
   Starting population at $t = 0$: $P = 30 \times 1.5^0 = 30 \times 1 = 30$ rabbits. After 3 years: $P = 30 \times 1.5^3 = 30 \times 3.375 = 101.25$, so about 101 rabbits.
   Achieved: applies an exponential model at given values.

8. A line passes through $(0, 4)$ and $(2, 10)$. Find its equation in the form $y = mx + c$.
   Gradient $m = (10 - 4)/(2 - 0) = 6/2 = 3$. The y-intercept is 4 (point at $x = 0$). So $y = 3x + 4$.
   Achieved: forms a linear equation from two points.

9. A drone is launched from a 5 m platform. Its height (m) after t seconds is $h = -5t^2 + 20t + 5$. Find the maximum height reached and when it hits the ground.
   Maximum at the vertex. Axis of symmetry: $t = -b/(2a) = -20/(2 \times -5) = 2$ s. Max height: $h = -5(2)^2 + 20(2) + 5 = -20 + 40 + 5 = 25$ m. Hits ground when $h = 0$: $-5t^2 + 20t + 5 = 0$, divide by -5: $t^2 - 4t - 1 = 0$, $t = (4 \pm \sqrt{(16 + 4)})/2 = (4 \pm \sqrt{20})/2 = 2 \pm \sqrt{5}$. Taking the positive root: $t = 2 + 2.236 \approx 4.24$ s.
   Merit: links vertex and roots to a physical situation via multi-step work.

10. The graph of $y = x^2$ is transformed to $y = 2(x - 3)^2 - 4$. Describe the sequence of transformations, then state the coordinates of the new vertex and the y-intercept.
    From $y = x^2$: horizontal shift right 3 (from $x - 3$), vertical stretch by factor 2 (the $\times 2$), then vertical shift down 4 (the $- 4$). New vertex is $(3, -4)$. y-intercept: set $x = 0$: $y = 2(0 - 3)^2 - 4 = 2 \times 9 - 4 = 18 - 4 = 14$, so $(0, 14)$.
    Merit: connects multiple transformations and derives resulting features.

11. Two courier companies charge for a delivery. FastVan charges $8 plus $2 per km; QuickGo charges a flat $20. Write an equation for each, find the distance where the costs are equal, and state which is cheaper for a 10 km trip.
    FastVan: $C = 8 + 2d$. QuickGo: $C = 20$. Equal when $8 + 2d = 20$, so $2d = 12$, $d = 6$ km. For 10 km FastVan costs $8 + 2 \times$

10 = $28, which is more than QuickGo's $20, so QuickGo is cheaper for 10 km (FastVan is cheaper below 6 km).
Merit: connects two representations to compare and interpret an intersection.

12. A parabola has x-intercepts at $x = -2$ and $x = 6$ and passes through $(2, -32)$. Find its equation in the form $y = ax^2 + bx + c$, and justify why the vertex must lie on $x = 2$.
With roots -2 and 6: $y = a(x + 2)(x - 6)$. Substitute $(2, -32)$: $-32 = a(2 + 2)(2 - 6) = a(4)(-4) = -16a$, so $a = 2$. Then $y = 2(x + 2)(x - 6) = 2(x^2 - 4x - 12) = 2x^2 - 8x - 24$. The axis of symmetry is midway between the roots: $x = (-2 + 6)/2 = 2$, so the vertex lies on $x = 2$, which is also why the given point $(2, -32)$ is exactly the vertex.
Excellence: reconstructs and justifies a model from features, recognising the point as the vertex.

13. An exponential model $y = a \cdot b^x$ passes through $(0, 12)$ and $(2, 3)$. Find a and b, and explain what value of b tells you about the situation and the behaviour of the graph.
At $x = 0$: $y = a \cdot b^0 = a = 12$. At $x = 2$: $3 = 12 \cdot b^2$, so $b^2 = 3/12 = 1/4$, $b = 1/2$ (taking the positive root). Model: $y = 12 \times (1/2)^x$. Since $0 < b < 1$ the quantity is decaying, it halves for every increase of 1 in x. The graph decreases toward the horizontal asymptote $y = 0$ but never reaches it, so y stays positive.
Excellence: solves for parameters and generalises the meaning of the base.

14. Prove that the line $y = x + k$ is tangent to the parabola $y = x^2$ only when $k = -1/4$, and state the point of tangency.
At intersection: $x^2 = x + k$, so $x^2 - x - k = 0$. A tangent means exactly one solution, i.e. the discriminant is zero: $b^2 - 4ac = (-1)^2 - 4(1)(-k) = 1 + 4k = 0$, giving $k = -1/4$. Then $x^2 - x + 1/4 = 0$, $(x - 1/2)^2 = 0$, so $x = 1/2$ and $y = (1/2)^2 = 1/4$. Point of tangency is $(1/2, 1/4)$.
Excellence: uses the discriminant condition to prove tangency and generalise.