NCEA Level 2 · AS 91258: Sequences & Series in Action

Name: _____ Date: _____

This standard is all about spotting patterns that grow by adding (arithmetic) or by multiplying (geometric), then using formulae to solve real problems, including money and growth. Work through each section in order and check your reasoning against the worked answers.

Revision summary

- Arithmetic sequence: constant difference d. nth term: $a_n = a + (n-1)d$. Sum: $S_n = n/2(2a + (n-1)d)$ or $n/2(a + l)$, where l is the last term.
- Geometric sequence: constant ratio r. nth term: $a_n = a \cdot r^{n-1}$. Sum: $S_n = a(r^n - 1)/(r - 1)$.
- Sum to infinity (only when $-1 < r < 1$): $S\infty = a/(1 - r)$.
- Sigma notation: Σ from k=1 to n of f(k) means add f(1) + f(2) + … + f(n). Read off the first term, the rule, and how many terms.
- Growth/decay: multiply by (1 + rate) each period → geometric. A quantity increasing $x each period → arithmetic.
- Compound interest amount after n periods: $A = P(1 + i)^n$. To solve for n, use logarithms: $n = \log(\text{target}/P) \div \log(1 + i)$.
- To decide arithmetic vs geometric: check if consecutive terms have a common difference (subtract) or a common ratio (divide).
- Solving S_n = a total often gives a quadratic in n, reject any negative or non-whole solution.

Key skills

1. An arithmetic sequence is 5, 8, 11, 14, … Find the 20th term.

2. Find the sum of the first 15 terms of the arithmetic series 4 + 7 + 10 + …

3. A geometric sequence is 3, 6, 12, 24, … Find the 8th term.

4. Evaluate Σ from k=1 to 5 of (2k + 1).

Achieved

5. A gym in Avondale charges $40 in the first month, then increases the monthly fee by $6 each month. How much is the fee in the 12th month?

6. A fish population in a lake starts at 500 and increases by 8% each year. Estimate the population after 6 years, to the nearest whole fish.

7. Evaluate Σ from k=1 to 10 of (3k − 2).

8. Mia saves $50 in her first week, then adds $5 more than the week before every week. How much has she saved in total after 12 weeks?

Merit

9. In an arithmetic sequence the 4th term is 23 and the 9th term is 48. Find the first term and common difference, then the sum of the first 20 terms.

10. A ball is dropped from a height of 2 m. Each bounce reaches 70% of the previous height. Find the total vertical distance the ball travels before coming to rest.

11. Ravi invests $2000 at 5% p.a. compound interest. Using logarithms, find the least whole number of years for the investment to first exceed $3000.

Excellence

12. Prove that the sum of the first n odd numbers, $1 + 3 + 5 + \ldots + (2n - 1)$, always equals n^2.

13. A geometric series has the sum of its first two terms equal to 12 and a sum to infinity of 18. Find the possible values of a and r, and justify that the series converges.

14. A concert hall has rows of seats where the first row has 10 seats and each row has 2 more seats than the row in front. The hall holds a total of 1170 seats. Find the number of rows, and explain why only one solution is valid.