NCEA Level 2 · AS 91258: Sequences & Series in Action

Name: _____ Date: _____

This standard is all about spotting patterns that grow by adding (arithmetic) or by multiplying (geometric), then using formulae to solve real problems, including money and growth. Work through each section in order and check your reasoning against the worked answers.

Revision summary

- Arithmetic sequence: constant difference d. nth term: $a_n = a + (n-1)d$. Sum: $S_n = n/2(2a + (n-1)d)$ or $n/2(a + l)$, where l is the last term.
- Geometric sequence: constant ratio r. nth term: $a_n = a \cdot r^{n-1}$. Sum: $S_n = a(r^n - 1)/(r - 1)$.
- Sum to infinity (only when $-1 < r < 1$): $S\infty = a/(1 - r)$.
- Sigma notation: Σ from k=1 to n of f(k) means add f(1) + f(2) + … + f(n). Read off the first term, the rule, and how many terms.
- Growth/decay: multiply by (1 + rate) each period → geometric. A quantity increasing $x each period → arithmetic.
- Compound interest amount after n periods: $A = P(1 + i)^n$. To solve for n, use logarithms: $n = \log(\text{target}/P) \div \log(1 + i)$.
- To decide arithmetic vs geometric: check if consecutive terms have a common difference (subtract) or a common ratio (divide).
- Solving S_n = a total often gives a quadratic in n, reject any negative or non-whole solution.

Key skills

1. An arithmetic sequence is 5, 8, 11, 14, … Find the 20th term.

2. Find the sum of the first 15 terms of the arithmetic series 4 + 7 + 10 + …

3. A geometric sequence is 3, 6, 12, 24, … Find the 8th term.

4. Evaluate Σ from k=1 to 5 of (2k + 1).

Achieved

5. A gym in Avondale charges $40 in the first month, then increases the monthly fee by $6 each month. How much is the fee in the 12th month?

6. A fish population in a lake starts at 500 and increases by 8% each year. Estimate the population after 6 years, to the nearest whole fish.

7. Evaluate Σ from k=1 to 10 of (3k − 2).

8. Mia saves $50 in her first week, then adds $5 more than the week before every week. How much has she saved in total after 12 weeks?

Merit

9. In an arithmetic sequence the 4th term is 23 and the 9th term is 48. Find the first term and common difference, then the sum of the first 20 terms.

10. A ball is dropped from a height of 2 m. Each bounce reaches 70% of the previous height. Find the total vertical distance the ball travels before coming to rest.

11. Ravi invests $2000 at 5% p.a. compound interest. Using logarithms, find the least whole number of years for the investment to first exceed $3000.

Excellence

12. Prove that the sum of the first n odd numbers, $1 + 3 + 5 + \ldots + (2n - 1)$, always equals n^2.

13. A geometric series has the sum of its first two terms equal to 12 and a sum to infinity of 18. Find the possible values of a and r, and justify that the series converges.

14. A concert hall has rows of seats where the first row has 10 seats and each row has 2 more seats than the row in front. The hall holds a total of 1170 seats. Find the number of rows, and explain why only one solution is valid.

Answer key (for parents and tutors)

1. An arithmetic sequence is 5, 8, 11, 14, … Find the 20th term.
   $a = 5$, $d = 3$. $a_{20} = a + 19d = 5 + 19\times3 = 5 + 57 = 62$.
   Achieved fluency: substitutes into $a_n = a + (n-1)d$.

2. Find the sum of the first 15 terms of the arithmetic series $4 + 7 + 10 + \ldots$
   $a = 4$, $d = 3$, $n = 15$. $S_{15} = 15/2(2\times4 + 14\times3) = 15/2(8 + 42) = 15/2 \times 50 = 375$.
   Achieved fluency: applies the arithmetic sum formula.

3. A geometric sequence is 3, 6, 12, 24, … Find the 8th term.
   $a = 3$, $r = 2$. $a_8 = a\cdot r^7 = 3 \times 2^7 = 3 \times 128 = 384$.
   Achieved fluency: applies $a_n = a\cdot r^{n-1}$.

4. Evaluate Σ from k=1 to 5 of $(2k + 1)$.
   Terms: k=1→3, k=2→5, k=3→7, k=4→9, k=5→11. Sum = 3 + 5 + 7 + 9 + 11 = 35.
   Achieved fluency: expands and sums sigma notation.

5. A gym in Avondale charges $40 in the first month, then increases the monthly fee by $6 each month. How much is the fee in the 12th month?
   Arithmetic: $a = 40$, $d = 6$. $a_{12} = 40 + 11\times6 = 40 + 66 = \$106$.
   Achieved: selects and applies $a_n = a + (n-1)d$ in context.

6. A fish population in a lake starts at 500 and increases by 8% each year. Estimate the population after 6 years, to the nearest whole fish.
   Geometric growth: multiply by 1.08 each year. After 6 years: $500 \times 1.08^6 = 500 \times 1.586874\ldots = 793.4 \approx 793$ fish.
   Achieved: applies compound growth with the multiplier 1.08.

7. Evaluate Σ from k=1 to 10 of $(3k - 2)$.
   This is arithmetic: first term (k=1) = 1, last term (k=10) = 28, n = 10. Sum = 10/2(1 + 28) = 5 × 29 = 145.
   Achieved: recognises sigma as an arithmetic series and sums it.

8. Mia saves $50 in her first week, then adds $5 more than the week before every week. How much has she saved in total after 12 weeks?
   Arithmetic: $a = 50$, $d = 5$, $n = 12$. $S_{12} = 12/2(2\times50 + 11\times5) = 6(100 + 55) = 6 \times 155 = \$930$.
   Achieved: applies the arithmetic sum formula to a savings context.

9. In an arithmetic sequence the 4th term is 23 and the 9th term is 48. Find the first term and common difference, then the sum of the first 20 terms.
   $d = (48 - 23)/(9 - 4) = 25/5 = 5$. $a = a_4 - 3d = 23 - 15 = 8$. $S_{20} = 20/2(2\times8 + 19\times5) = 10(16 + 95) = 10 \times 111 = 1110$.
   Merit: links two term equations to find a and d, then chains into the sum.

10. A ball is dropped from a height of 2 m. Each bounce reaches 70% of the previous height. Find the total vertical distance the ball travels before coming to rest.
    Initial drop = 2 m. First bounce up = $2\times0.7 = 1.4$ m, and the up-heights form a geometric series with $a = 1.4$, $r = 0.7$. Since $-1 < r < 1$: $S_\infty = 1.4/(1 - 0.7) = 1.4/0.3 = 4.667$ m. Each bounce goes up then down, so bounce distance = $2 \times 4.667 = 9.333$ m. Total = $2 + 9.333 = 11.33$ m (2 dp).
    Merit: links geometric sum to infinity with the up-and-down structure of the motion.

11. Ravi invests $2000 at 5% p.a. compound interest. Using logarithms, find the least whole number of years for the investment to first exceed $3000.
    Need $2000 \times 1.05^n > 3000 \rightarrow 1.05^n > 1.5$. Take logs: $n > \log(1.5)/\log(1.05) = 0.17609/0.02119 = 8.31$. So n = 9 years.
    Merit: connects compound growth to logarithms and interprets the answer as a whole year.

12. Prove that the sum of the first n odd numbers, $1 + 3 + 5 + \ldots + (2n - 1)$, always equals n^2.
    The odd numbers form an arithmetic series with $a = 1$, $d = 2$, and nth term $2n - 1$. Sum = $n/2(a + l) = n/2(1 + (2n - 1)) = n/2 \times 2n = n^2$. Since this holds for the general n using the arithmetic sum formula, the sum of the first n odd numbers is exactly n^2 for every positive integer n.
    Excellence: generalises and justifies a result algebraically for all n.

13. A geometric series has the sum of its first two terms equal to 12 and a sum to infinity of 18. Find the possible values of a and r, and justify that the series converges.
$S_\infty = a/(1 - r) = 18$, so $a = 18(1 - r)$. First two terms: $a + ar = a(1 + r) = 12$. Substitute: $18(1 - r)(1 + r) = 12 \rightarrow 18(1 - r^2) = 12 \rightarrow 1 - r^2 = 2/3 \rightarrow r^2 = 1/3 \rightarrow r = \pm 1/\sqrt{3} \approx \pm 0.577$. Both satisfy $|r| < 1$, so the series converges (each term shrinks in size). If $r = 0.577$, $a = 18(1 - 0.577) = 7.61$; if $r = -0.577$, $a = 18(1 + 0.577) = 28.39$.
Excellence: solves a two-condition system, finds both cases and justifies convergence.

14. A concert hall has rows of seats where the first row has 10 seats and each row has 2 more seats than the row in front. The hall holds a total of 1170 seats. Find the number of rows, and explain why only one solution is valid.
Arithmetic: $a = 10$, $d = 2$. $S_n = n/2(2 \times 10 + (n - 1)2) = n/2(20 + 2n - 2) = n/2(18 + 2n) = n(9 + n) = n^2 + 9n$. Set equal to 1170: $n^2 + 9n - 1170 = 0$. Discriminant $= 81 + 4680 = 4761 = 69^2$. $n = (-9 \pm 69)/2$, giving $n = 30$ or $n = -39$. The number of rows must be a positive whole number, so $n = -39$ is rejected. There are 30 rows.
Excellence: forms and solves a quadratic from a sum, and justifies rejecting the invalid root.