

NCEA Level 2 · AS 91259: Trig: Rules, Bearings & Radians

Name: _____

Date: _____

Kia ora! This standard is about picking the right trig tool, sine rule, cosine rule, area formula, or radian measure, and using it to crack real problems, including bearings and 3D. Work through each section in order; the answers show the full working a marker wants to see.

Revision summary

- Sine rule: $a/\sin A = b/\sin B = c/\sin C$. Use when you have a side–opposite–angle pair (find a side or an angle).
- Cosine rule: $c^2 = a^2 + b^2 - 2ab \cdot \cos C$. Use for SAS (find a side) or SSS (rearrange: $\cos C = (a^2 + b^2 - c^2)/(2ab)$ to find an angle).
- Area of a triangle = $\frac{1}{2} ab \cdot \sin C$ (two sides and the included angle). Heron's: $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$, $s = (a+b+c)/2$.
- Bearings are measured clockwise from north, written as three digits (e.g. 072°). Sketch a north line at each point and find the interior angle before applying a rule.
- Radians: $180^\circ = \pi$ rad. Convert: $\text{degrees} \times \pi/180 = \text{radians}$; $\text{radians} \times 180/\pi = \text{degrees}$.
- Arc length $L = r\theta$ and sector area $A = \frac{1}{2}r^2\theta$, with θ in radians.
- Segment area = $\frac{1}{2}r^2(\theta - \sin\theta)$; chord length = $2r \cdot \sin(\theta/2)$, θ in radians.
- Ambiguous case (SSA): $\sin B = b \cdot \sin A/a$ may give two answers (B and $180^\circ - B$), check each keeps $A+B < 180^\circ$.

Key skills

1. In triangle ABC, angle $A = 40^\circ$, angle $B = 65^\circ$ and side $a = 12$ cm. Find side b .
2. Two sides of a triangle are 8 m and 11 m with an included angle of 52° . Find the third side.
3. A triangle has sides 9 cm and 14 cm with an included angle of 38° . Find its area.
4. A sector has radius 10 cm and central angle 1.2 radians. Find the arc length and the sector area.

Achieved

5. A tramper leaves the hut and walks 8 km on a bearing of 120° . How far south of the hut is she?
6. In triangle ABC, side $a = 15$ cm is opposite angle A , and side $b = 10$ cm is opposite angle $B = 35^\circ$. Find angle A .
7. A triangular paddock has sides 7 m, 9 m and 12 m. Find the size of its largest angle.
8. An arc of a circle of radius 6 cm has length 15 cm. Find the central angle in radians, then in degrees.

Merit

9. A yacht sails 12 km on a bearing of 050° , then turns and sails 8 km on a bearing of 140° . Find its straight-line distance from the start, and the bearing it must steer to return directly to the start.
10. A triangular garden has sides 18 m, 22 m and 26 m. Find its area.
11. In triangle ABC, side $a = 9$ cm is opposite angle $A = 40^\circ$, and side $b = 12$ cm is opposite angle B. Show that there are two possible triangles and find both values of B.

Excellence

12. From point A on flat ground the angle of elevation to the top of a flagpole is 28° . Walking 40 m directly toward the base to point B, the angle of elevation becomes 45° . Find the height of the flagpole, justifying your method.
13. Prove that the area of any triangle can be written as $\text{Area} = (a^2 \cdot \sin B \cdot \sin C) / (2 \cdot \sin A)$, where a, b, c are sides opposite angles A, B, C.
14. A circle has radius 8 cm. A chord of the circle is 10 cm long. Find the area of the minor segment cut off by the chord.

Answer key (for parents and tutors)

- In triangle ABC, angle A = 40° , angle B = 65° and side a = 12 cm. Find side b.
Sine rule: $b/\sin B = a/\sin A$, so $b = a \cdot \sin B / \sin A = 12 \cdot \sin 65^\circ / \sin 40^\circ = 12 \cdot 0.9063 / 0.6428 = 16.9$ cm.
Achieved: selects and applies the sine rule to find a side.
- Two sides of a triangle are 8 m and 11 m with an included angle of 52° . Find the third side.
Cosine rule: $c^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cdot \cos 52^\circ = 64 + 121 - 176 \cdot 0.6157 = 185 - 108.4 = 76.64$, so $c = \sqrt{76.64} = 8.75$ m.
Achieved: applies the cosine rule (SAS) for a side.
- A triangle has sides 9 cm and 14 cm with an included angle of 38° . Find its area.
Area = $\frac{1}{2} \cdot 9 \cdot 14 \cdot \sin 38^\circ = 63 \cdot 0.6157 = 38.8$ cm².
Achieved: applies the $\frac{1}{2}ab \cdot \sin C$ area formula.
- A sector has radius 10 cm and central angle 1.2 radians. Find the arc length and the sector area.
Arc L = $r\theta = 10 \cdot 1.2 = 12$ cm. **Sector area** = $\frac{1}{2}r^2\theta = \frac{1}{2} \cdot 10^2 \cdot 1.2 = 60$ cm².
Achieved: applies arc length and sector area formulae in radians.
- A trapper leaves the hut and walks 8 km on a bearing of 120° . How far south of the hut is she?
North component = $8 \cdot \cos 120^\circ = 8 \cdot (-0.5) = -4$, i.e. 4 km. The negative shows the movement is southward, so she is 4 km south of the hut.
Achieved: resolves a bearing into a directional component.
- In triangle ABC, side a = 15 cm is opposite angle A, and side b = 10 cm is opposite angle B = 35° . Find angle A.
Sine rule: $\sin A/a = \sin B/b$, so $\sin A = a \cdot \sin B / b = 15 \cdot \sin 35^\circ / 10 = 15 \cdot 0.5736 / 10 = 0.8604$. $A = \sin^{-1}(0.8604) = 59.4^\circ$.
Achieved: applies the sine rule to find an angle.
- A triangular paddock has sides 7 m, 9 m and 12 m. Find the size of its largest angle.
The largest angle is opposite the longest side (12 m). $\cos C = (7^2 + 9^2 - 12^2) / (2 \cdot 7 \cdot 9) = (49 + 81 - 144) / 126 = -14 / 126 = -0.1111$. $C = \cos^{-1}(-0.1111) = 96.4^\circ$.
Achieved: applies the cosine rule (SSS) to find an angle.
- An arc of a circle of radius 6 cm has length 15 cm. Find the central angle in radians, then in degrees.
L = $r\theta$, so $\theta = L/r = 15/6 = 2.5$ radians. In degrees: $2.5 \cdot 180/\pi = 143.2^\circ$.
Achieved: uses arc length formula and converts radians to degrees.
- A yacht sails 12 km on a bearing of 050° , then turns and sails 8 km on a bearing of 140° . Find its straight-line distance from the start, and the bearing it must steer to return directly to the start.
Break into east/north components. Leg 1: E = $12 \sin 50^\circ = 9.192$, N = $12 \cos 50^\circ = 7.713$. **Leg 2:** E = $8 \sin 140^\circ = 5.142$, N = $8 \cos 140^\circ = -6.128$. **Totals:** E = 14.334, N = 1.585. **Distance** = $\sqrt{(14.334^2 + 1.585^2)} = \sqrt{(205.46 + 2.51)} = \sqrt{207.97} = 14.4$ km. **Bearing from start to yacht** = $\tan^{-1}(E/N) = \tan^{-1}(14.334/1.585) = 83.7^\circ$, i.e. 084° . To return, steer the reverse bearing: $084^\circ + 180^\circ = 264^\circ$.
Merit: links two-leg bearings via components and reverses the bearing to solve a multi-step problem.
- A triangular garden has sides 18 m, 22 m and 26 m. Find its area.
Find the angle between the 18 m and 22 m sides (opposite 26 m): $\cos C = (18^2 + 22^2 - 26^2) / (2 \cdot 18 \cdot 22) = (324 + 484 - 676) / 792 = 132 / 792 = 0.1667$, so $C = 80.4^\circ$. **Area** = $\frac{1}{2} \cdot 18 \cdot 22 \cdot \sin 80.4^\circ = 198 \cdot 0.9862 = 195$ m². (Check with Heron's: $s = 33$, **Area** = $\sqrt{33 \cdot 15 \cdot 11 \cdot 7} = \sqrt{38115} = 195$ m².)
Merit: connects the cosine rule to the area formula (or Heron's) across two steps.
- In triangle ABC, side a = 9 cm is opposite angle A = 40° , and side b = 12 cm is opposite angle B. Show that there are two possible triangles and find both values of B.
Sine rule: $\sin B = b \cdot \sin A / a = 12 \cdot \sin 40^\circ / 9 = 12 \cdot 0.6428 / 9 = 0.8570$. $B = \sin^{-1}(0.8570) = 59.0^\circ$, or the obtuse partner $B = 180^\circ - 59.0^\circ = 121.0^\circ$. **Check both:** with $B = 59.0^\circ$, $A + B = 99.0^\circ < 180^\circ$ ✓; with $B = 121.0^\circ$, $A + B = 161.0^\circ < 180^\circ$ ✓. Since $b > a$, the side opposite the larger angle is longer, so both are geometrically valid, two triangles exist.
Merit: recognises and justifies the ambiguous (SSA) case with a validity check.
- From point A on flat ground the angle of elevation to the top of a flagpole is 28° . Walking 40 m directly toward the base to point B, the angle of elevation becomes 45° . Find the height of the flagpole, justifying your method.

Let h be the height and d the horizontal distance from B to the base. At B the elevation is 45° , so $\tan 45^\circ = h/d = 1$, giving $d = h$. At A the distance to the base is $d + 40$, so $\tan 28^\circ = h/(d + 40) = h/(h + 40)$. Thus $h = (h + 40)\tan 28^\circ = 0.5317(h + 40) = 0.5317h + 21.27$. So $h - 0.5317h = 21.27$, $0.4683h = 21.27$, $h = 45.4$ m. The 45° angle lets us equate horizontal distance and height, which unlocks a single equation in h .

Excellence: models a 3D-style elevation problem with two unknowns and justifies the key simplification.

13. Prove that the area of any triangle can be written as $\text{Area} = (a^2 \cdot \sin B \cdot \sin C) / (2 \cdot \sin A)$, where a, b, c are sides opposite angles A, B, C .

Start from $\text{Area} = \frac{1}{2} \cdot a \cdot b \cdot \sin C$. By the sine rule, $b/\sin B = a/\sin A$, so $b = a \cdot \sin B / \sin A$. Substitute: $\text{Area} = \frac{1}{2} \cdot a \cdot (a \cdot \sin B / \sin A) \cdot \sin C = (a^2 \cdot \sin B \cdot \sin C) / (2 \cdot \sin A)$. This holds for any triangle because both the area formula and the sine rule apply to all triangles.

Excellence: derives a general formula by combining two standard results algebraically.

14. A circle has radius 8 cm. A chord of the circle is 10 cm long. Find the area of the minor segment cut off by the chord.
The chord relates to the central angle θ by $\text{chord} = 2r \cdot \sin(\theta/2)$: $10 = 2 \cdot 8 \cdot \sin(\theta/2)$, so $\sin(\theta/2) = 10/16 = 0.625$, giving $\theta/2 = 0.6751$ rad and $\theta = 1.350$ rad. Segment area $= \frac{1}{2}r^2(\theta - \sin \theta) = \frac{1}{2} \cdot 8^2 \cdot (1.350 - \sin 1.350) = 32 \cdot (1.350 - 0.9757) = 32 \cdot 0.3746 = 12.0$ cm².

Excellence: chains chord $\rightarrow$ central angle $\rightarrow$ segment area in an unfamiliar multi-step radian problem.