NCEA Level 2 · AS 91261: Algebra Toolkit

Name: _____ Date: _____

Kia ora! This standard is all about the algebra machinery: simplifying with indices, factorising, and solving quadratic, exponential and log equations. Work through each section in order, check every answer against the worked solution and note the grade level you're aiming for.

Revision summary

- Index laws: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^0 = 1$, $a^{-n} = 1/a^n$, $a^{(1/n)} = \sqrt[n]{a}$.
- Logarithm definition: if $a^x = b$ then $\log_a b = x$. Log laws: $\log(xy) = \log x + \log y$, $\log(x/y) = \log x - \log y$, $\log(x^n) = n \log x$.
- Quadratic formula: for $ax^2 + bx + c = 0$, $x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$.
- Discriminant $\Delta = b^2 - 4ac$: $\Delta > 0$ two real roots, $\Delta = 0$ one repeated root, $\Delta < 0$ no real roots.
- Factorising: common factor first, then quadratics $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$; difference of two squares $a^2 - b^2 = (a - b)(a + b)$.
- Exponential equations: write both sides with the same base, then equate the powers.
- Log equations: combine to a single log, rewrite in index form, then solve, and always check answers against the domain (you can only take logs of positive numbers).

Key skills

1. Simplify $(3a^2b)^3 \times 2ab^2$.

2. Factorise $x^2 + 3x - 10$.

3. Solve $2^x = 32$.

4. Evaluate $\log_3 81$.

Achieved

5. Solve $x^2 - 5x + 6 = 0$ by factorising.

6. Simplify $(12x^5y^3) \div (4x^2y^5)$.

7. Use the quadratic formula to solve $2x^2 + 3x - 4 = 0$. Give exact answers.

8. Solve 3^(x+1) = 81.

Merit

9. A rectangular deck at a bach in Raglan has a length 3 m longer than its width. The area of the deck is 40 m². Form an equation and find the dimensions of the deck.

10. Write $3/(x-2) + 2/(x+1)$ as a single simplified fraction.

11. Solve $9^x = 27^(x-2)$.

Excellence

12. For what values of k does the equation $x^2 + kx + 4 = 0$ have two distinct real roots? Justify your answer.

13. Solve $\log_2 x + \log_2(x-2) = 3$, showing why any invalid solution must be rejected.

14. Prove that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.