NCEA Level 2 · AS 91262: Calculus Methods

Name: _____ Date: _____

This standard is all about differentiation, integration, and using them to solve gradient, tangent, optimisation and motion problems. Work through the sections in order, build fluency first, then push into the Merit and Excellence reasoning.

Revision summary

- Differentiation: if $y = ax^n$ then $dy/dx = anx^{n-1}$. The derivative gives the gradient function.
- Gradient of a curve at $x = a$ is found by substituting a into dy/dx. Tangent line: $y - y_1 = m(x - x_1)$.
- Anti-differentiation (integration): $\int ax^n\,dx = ax^{n+1}/(n+1) + c$ $(n \neq -1)$. Always add $+ c$ for an indefinite integral; find c using a known point.
- Increasing where $dy/dx > 0$, decreasing where $dy/dx < 0$. Turning points where $dy/dx = 0$.
- Second derivative test: $d^2y/dx^2 > 0 \rightarrow$ minimum, $d^2y/dx^2 < 0 \rightarrow$ maximum.
- Kinematics: differentiate displacement $s \rightarrow$ velocity $v \rightarrow$ acceleration a. Integrate $a \rightarrow v \rightarrow s$. At rest means $v = 0$.
- Optimisation: write the quantity as a function of one variable, differentiate, set derivative $= 0$, solve, and check it is a max or min.
- Units: gradient/rate = change in y per unit x; velocity in m s^{-1}, acceleration in m s^{-2}.

Key skills

1. Differentiate $y = 4x^3 - 5x^2 + 7x - 2$.

2. Find $\int (6x^2 + 4x - 3)\,dx$.

3. Find the gradient of the curve $y = x^2 - 6x + 5$ at the point where $x = 4$.

4. A ball's height is $h = 20t - 5t^2$ metres. Find its velocity function.

Achieved

5. Find the equation of the tangent to $y = x^2 - 3x + 1$ at the point where $x = 2$.

6. The gradient function of a curve is $dy/dx = 3x^2 - 4x$. The curve passes through $(1, 2)$. Find y as a function of x.

7. Find the coordinates of the turning point of $y = x^2 - 8x + 10$.

8. A drone rises so its height is $h = t^3 - 6t^2 + 9t$ metres, where t is in seconds. Find its velocity when $t = 1$ s.

Merit

9. A rectangular vegetable garden in Avondale is to be fenced against a straight wall, so only three sides need fencing. There are 60 m of fencing available. Find the dimensions that give the maximum area.

10. A curve has gradient function $dy/dx = 6x - 12$. Find the x-values where the curve is decreasing, and find where the curve has a turning point.

11. A particle moves so its velocity is $v = 3t^2 - 12t + 9$ m s^{-1}. It starts at position $s = 5$ m when $t = 0$. Find its displacement function and the times when it is at rest.

12. The tangent to the curve $y = x^2 + kx - 4$ at the point where $x = 1$ has gradient 5. Find k, then find the equation of that tangent.

Excellence

13. A cylindrical rainwater tank (no lid) is to hold a fixed volume V. Show that the surface area of metal used is least when the height equals the radius.

14. A curve $y = ax^3 + bx^2$ has a turning point at $(2, -4)$. Find a and b, justifying your reasoning.

15. A ball is thrown upward with height $h = ut - 5t^2$ metres (u is the initial speed in m s^{-1}). Prove that the maximum height reached is $u^2/20$ metres, regardless of u.