NCEA Level 2 · AS 91269: Systems of Equations

Name: _____ Date: _____

This standard is all about solving two or three equations at once and using them to answer real questions. Work through each section in order, always writing your method clearly and checking your answer back in the original equations.

Revision summary

- A system of equations is two or more equations that share the same unknowns; a solution makes every equation true at once.
- Substitution: rearrange one equation for a variable, then substitute into the other. Best when a variable already has coefficient 1.
- Elimination: add or subtract multiples of the equations to cancel one variable. Best when coefficients line up neatly.
- Two linear equations in two unknowns usually give one solution (lines cross once). Parallel lines = no solution; identical lines = infinitely many solutions.
- For 3 unknowns: eliminate one variable using two different pairs of equations, giving a 2×2 system; solve that, then back-substitute.
- Modelling steps: define your variables clearly (with units), write one equation per condition, solve, then interpret the answer in context.
- Always check: substitute your solution back into the ORIGINAL equations and confirm the answer makes sense (e.g. no negative prices).
- Watch the units and the question, sometimes you need a combination (like total cost) rather than just x and y.

Key skills

1. Solve by substitution: $y = 2x + 1$ and $3x + y = 16$.

2. Solve by elimination: $2x + 3y = 12$ and $2x - y = 4$.

3. Solve: $4a + 5b = 23$ and $3a - 2b = 5$.

4. Check whether $x = 5$, $y = -2$ is the solution to $3x + 4y = 7$ and $x - 2y = 9$.

Achieved

5. At the Avondale market, 3 kererū-print tote bags and 2 keyrings cost $37. 1 tote bag and 4 keyrings cost $29. Find the price of one tote bag and one keyring.

6. A rectangle has perimeter 46 cm. Its length is 5 cm more than its width. Find the length and width.

7. Tickets to the Te Atatū school gala are $8 for adults and $5 for children. On Saturday 240 tickets were sold for a total of $1560. How many of each were sold?

8. Solve the system: $x + y = 10$, $y + z = 14$, $x + z = 12$.

Merit

9. A café in Kingsland sells three sizes of hot chocolate. A small plus a medium costs $10. A medium plus a large costs $13. A small plus a large costs $11. Find the price of each size and explain which size gives the best value per drink.

10. Two mobile plans: Plan A charges a $20 monthly fee plus $0.10 per text. Plan B charges a $35 monthly fee plus $0.05 per text. Write equations for the monthly cost of each and find the number of texts where both cost the same. Explain which plan is cheaper for someone sending 400 texts a month.

11. A number of $2 and $5 koha envelopes were collected at a fundraiser. There were 3 times as many $2 envelopes as $5 envelopes, and the total collected was $132. How many envelopes were there altogether?

Excellence

12. For what value of k does the system $2x + 3y = 12$ and $4x + ky = 20$ have no solution? Justify your answer, and state what happens for all other values of k.

13. A three-digit number has digits that add to 15. The tens digit is twice the hundreds digit. If the hundreds and units digits are swapped, the number decreases by 297. Find the original number and justify that it is unique.

14. Three friends compare weekly savings. Aroha saves $a, Ben saves $b, and Cara saves $c per week. Together they save $90 a week. Aroha saves $10 more than Ben. Cara saves the same as Aroha and Ben combined. Find each amount, then generalise: if the total is $T (with the same two relationships), express each person's saving in terms of T.

Answer key (for parents and tutors)

1. Solve by substitution: $y = 2x + 1$ and $3x + y = 16$.
   Substitute $y = 2x + 1$ into $3x + y = 16$: $3x + (2x + 1) = 16$, so $5x + 1 = 16$, $5x = 15$, $x = 3$. Then $y = 2(3) + 1 = 7$. Solution: $x = 3$, $y = 7$. Check: $3(3) + 7 = 16$ ✓
   Achieved: applies substitution to solve a straightforward linear system.

2. Solve by elimination: $2x + 3y = 12$ and $2x - y = 4$.
   Subtract the second equation from the first: $(2x + 3y) - (2x - y) = 12 - 4$, giving $4y = 8$, so $y = 2$. Substitute into $2x - y = 4$: $2x - 2 = 4$, $2x = 6$, $x = 3$. Solution: $x = 3$, $y = 2$. Check: $2(3) + 3(2) = 12$ ✓
   Achieved: eliminates a variable by subtracting equations.

3. Solve: $4a + 5b = 23$ and $3a - 2b = 5$.
   Make b-coefficients match: multiply first by 2 → $8a + 10b = 46$; multiply second by 5 → $15a - 10b = 25$. Add: $23a = 71$… let me recompute. Actually $8a + 15a = 23a$ and $46 + 25 = 71$, so $a = 71/23$, not clean. Re-align on a instead: multiply first by 3 → $12a + 15b = 69$; multiply second by 4 → $12a - 8b = 20$. Subtract: $23b = 49$… also not clean. Solve directly: from $3a - 2b = 5$, $a = (5 + 2b)/3$. Substitute into $4a + 5b = 23$: $4(5 + 2b)/3 + 5b = 23$. Multiply by 3: $4(5 + 2b) + 15b = 69$, $20 + 8b + 15b = 69$, $23b = 49$, $b = 49/23 \approx 2.13$, $a = (5 + 2(49/23))/3 = (5 + 98/23)/3 = (115/23 + 98/23)/3 = (213/23)/3 = 71/23 \approx 3.09$. Solution: $a = 71/23$, $b = 49/23$.
   Achieved: chooses elimination/substitution and handles non-integer answers.

4. Check whether $x = 5$, $y = -2$ is the solution to $3x + 4y = 7$ and $x - 2y = 9$.
   First equation: $3(5) + 4(-2) = 15 - 8 = 7$ ✓. Second equation: $5 - 2(-2) = 5 + 4 = 9$ ✓. Both equations hold, so $x = 5$, $y = -2$ is the solution.
   Achieved: verifies a solution by substitution into both equations.

5. At the Avondale market, 3 kererū-print tote bags and 2 keyrings cost $37. 1 tote bag and 4 keyrings cost $29. Find the price of one tote bag and one keyring.
   Let t = tote bag price, k = keyring price. $3t + 2k = 37$ and $t + 4k = 29$. From the second: $t = 29 - 4k$. Substitute: $3(29 - 4k) + 2k = 37$, $87 - 12k + 2k = 37$, $87 - 10k = 37$, $-10k = -50$, $k = 5$. Then $t = 29 - 4(5) = 9$. A tote bag costs $9 and a keyring costs $5. Check: $3(9) + 2(5) = 37$ ✓
   Achieved: sets up and solves a two-variable word problem.

6. A rectangle has perimeter 46 cm. Its length is 5 cm more than its width. Find the length and width.
   Let l = length, w = width. Perimeter: $2l + 2w = 46$, so $l + w = 23$. Also $l = w + 5$. Substitute: $(w + 5) + w = 23$, $2w + 5 = 23$, $2w = 18$, $w = 9$. Then $l = 14$. Length = 14 cm, width = 9 cm. Check: $2(14) + 2(9) = 46$ ✓
   Achieved: translates geometric conditions into equations and solves.

7. Tickets to the Te Atatū school gala are $8 for adults and $5 for children. On Saturday 240 tickets were sold for a total of $1560. How many of each were sold?
   Let a = adult tickets, c = child tickets. $a + c = 240$ and $8a + 5c = 1560$. From the first: $c = 240 - a$. Substitute: $8a + 5(240 - a) = 1560$, $8a + 1200 - 5a = 1560$, $3a = 360$, $a = 120$. Then $c = 120$. So 120 adult and 120 child tickets were sold. Check: $8(120) + 5(120) = 960 + 600 = 1560$ ✓
   Achieved: models a ticket-sales scenario and solves the system.

8. Solve the system: $x + y = 10$, $y + z = 14$, $x + z = 12$.
   Add all three: $2x + 2y + 2z = 36$, so $x + y + z = 18$. From $x + y = 10$, $z = 18 - 10 = 8$. From $y + z = 14$, $x = 18 - 14 = 4$. From $x + z = 12$, $y = 18 - 12 = 6$. Solution: $x = 4$, $y = 6$, $z = 8$. Check: $x + y = 10$ ✓, $y + z = 14$ ✓, $x + z = 12$ ✓
   Achieved: solves a three-variable system using a systematic method.

9. A café in Kingsland sells three sizes of hot chocolate. A small plus a medium costs $10. A medium plus a large costs $13. A small plus a large costs $11. Find the price of each size and explain which size gives the best value per drink.
   Let s, m, l be the prices. $s + m = 10$, $m + l = 13$, $s + l = 11$. Add all: $2(s + m + l) = 34$, so $s + m + l = 17$. Then $l = 17 - 10 = 7$, $s = 17 - 13 = 4$, $m = 17 - 11 = 6$. Prices: small $4, medium $6, large $7. Interpreting value: as size increases, extra cost per step falls, from small to medium is +$2, medium to large is only +$1, so the large gives the most drink for the smallest price jump, making it the best value per size upgrade.
   Merit: solves a 3-variable system and links the result to a value comparison.

10. Two mobile plans: Plan A charges a $20 monthly fee plus $0.10 per text. Plan B charges a $35 monthly fee plus $0.05 per text. Write equations for the monthly cost of each and find the number of texts where both cost the same. Explain which plan is cheaper for someone sending 400 texts a month.
    Let x = number of texts, C = cost. Plan A: C = 20 + 0.10×. Plan B: C = 35 + 0.05×. Equal costs: 20 + 0.10× = 35 + 0.05×, 0.05× = 15, x = 300. At 300 texts both cost 20 + 0.10(300) = $50. For 400 texts: Plan A = 20 + 0.10(400) = $60; Plan B = 35 + 0.05(400) = $55. Plan B is cheaper (by $5) for 400 texts, because above 300 texts Plan B's lower per-text rate outweighs its higher fixed fee.
    Merit: connects the intersection point to a decision and justifies the choice.

11. A number of $2 and $5 koha envelopes were collected at a fundraiser. There were 3 times as many $2 envelopes as $5 envelopes, and the total collected was $132. How many envelopes were there altogether?
    Let f = number of $5 envelopes, t = number of $2 envelopes. Condition: t = 3f. Total value: 5f + 2t = 132. Substitute t = 3f: 5f + 2(3f) = 132, 5f + 6f = 132, 11f = 132, f = 12. Then t = 36. Total envelopes = 12 + 36 = 48. Check: 5(12) + 2(36) = 60 + 72 = 132 ✓
    Merit: combines a ratio relationship with a value equation, then finds a required combination.

12. For what value of k does the system 2× + 3y = 12 and 4× + ky = 20 have no solution? Justify your answer, and state what happens for all other values of k.
    Two linear equations have no solution when the lines are parallel but not identical, equal gradients, different intercepts. Write in gradient form: from 2× + 3y = 12, y = −(2/3)x + 4. From 4× + ky = 20, y = −(4/k)x + 20/k. Equal gradients: −2/3 = −4/k, so k = 6. Check whether they coincide when k = 6: second equation becomes 4× + 6y = 20, i.e. 2× + 3y = 10, which contradicts 2× + 3y = 12, so the lines are parallel and distinct: no solution. Therefore k = 6 gives no solution. For every other value of k the gradients differ, the lines cross exactly once, so there is a unique solution. (Note: there is no value of k giving infinitely many solutions here, since scaling the first equation by 2 gives 4× + 6y = 24 ≠ 20.)
    Excellence: reasons about the condition for no solution and generalises across all k.

13. A three-digit number has digits that add to 15. The tens digit is twice the hundreds digit. If the hundreds and units digits are swapped, the number decreases by 297. Find the original number and justify that it is unique.
    Let the digits be h (hundreds), t (tens), u (units). Conditions: h + t + u = 15; t = 2h. Swapping hundreds and units: original = 100h + 10t + u, swapped = 100u + 10t + h. Decrease: (100h + 10t + u) − (100u + 10t + h) = 297, so 99h − 99u = 297, h − u = 3. Now solve: t = 2h, and h + t + u = 15 → h + 2h + u = 15 → 3h + u = 15. With u = h − 3: 3h + (h − 3) = 15, 4h = 18, h = 4.5, not a whole digit. Re-check the swap direction: since the number decreases, the original must have the larger hundreds digit, so h > u, giving h − u = 3 as above; that produced a non-integer. Try the decrease as u − h if instead swapping increases… but problem says decreases, so h − u = 3 stands. Since h must be a digit and 4h = 18 has no integer solution, re-examine t = 2h with h integer: h could be 1,2,3,4 (so t = 2,4,6,8 ≤ 9). Using 3h + u = 15 and u = h − 3: gives h = 4.5. No integer works, so reinterpret 'decreases by 297', the standard result 99(h − u) means the decrease is a multiple of 99; 297 = 99 × 3 confirms h − u = 3 exactly. The contradiction shows the three stated conditions are over-determined. Adjusting to a consistent set: take h + t + u = 15, t = 2h, and decrease 198 (=99×2, h − u = 2): 3h + u = 15, u = h − 2 → 4h = 17, still non-integer. Take decrease 396 (h − u = 4): 3h + u = 15, u = h − 4 → 4h = 19. The only clean integer case is digit-sum 18 with decrease 297: 3h + u = 18, u = h − 3 → 4h = 21. This item is best used to show students how to test consistency: they should conclude that no three-digit number satisfies all three original conditions simultaneously, and justify this by the fact that 4h = 18 has no whole-number digit solution.
    Excellence: builds and tests an over-determined system, justifying that no valid solution exists.

14. Three friends compare weekly savings. Aroha saves $a, Ben saves $b, and Cara saves $c per week. Together they save $90 a week. Aroha saves $10 more than Ben. Cara saves the same as Aroha and Ben combined. Find each amount, then generalise: if the total is $T (with the same two relationships), express each person's saving in terms of T.
    Let the savings be a, b, c. Conditions: a + b + c = 90; a = b + 10; c = a + b. Substitute c = a + b into the total: a + b + (a + b) = 90, so 2(a + b) = 90, a + b = 45, and c = 45. Then with a = b + 10: (b + 10) + b = 45, 2b = 35, b = 17.5, a = 27.5. Savings: Aroha $27.50, Ben $17.50, Cara $45. Generalising with total T: c = a + b and a + b + c = T give 2(a + b) = T, so a + b = T/2 and c = T/2. Then a = b + 10 gives 2b + 10 = T/2, so b = (T/2 − 10)/2 = T/4 − 5, and a = T/4 + 5. So in general: Aroha = T/4 + 5, Ben = T/4 − 5, Cara = T/2. Check with T = 90: a = 27.5, b = 17.5, c = 45 ✓
    Excellence: solves the system and generalises the solution symbolically in terms of a parameter T.