NCEA Level 3 · AS 91575: Trig Methods & Periodic Models

Name: _____ Date: _____

Kia ora! This standard is all about solving trig equations (in radians), using compound and double-angle formulae, proving identities, and modelling things that repeat like tides and Ferris wheels. Work top to bottom, nail the Key skills first, then push into the graded questions.

Revision summary

- Radians: π rad = 180°. To convert degrees→radians ×π/180; radians→degrees ×180/π. Common: π/6=30°, π/4=45°, π/3=60°, π/2=90°.
- Compound angles: $\sin(A\pm B)=\sin A\cos B \pm \cos A\sin B$; $\cos(A\pm B)=\cos A\cos B \mp \sin A\sin B$.
- Double angles: $\sin 2\theta=2\sin\theta\cos\theta$; $\cos 2\theta=\cos^2\theta-\sin^2\theta=1-2\sin^2\theta=2\cos^2\theta-1$.
- General solutions: $\sin x=k \rightarrow x=\alpha+2n\pi$ or $(\pi-\alpha)+2n\pi$; $\cos x=k \rightarrow x=\pm\alpha+2n\pi$; $\tan x=k \rightarrow x=\alpha+n\pi$ (n is any integer).
- Pythagorean identity: $\sin^2\theta+\cos^2\theta=1$, so $\tan\theta=\sin\theta/\cos\theta$, $\cot\theta=\cos\theta/\sin\theta$.
- Model form $y=A\sin(B(t-C))+D$: amplitude=$|A|$, period=$2\pi/B$, D=vertical shift (midline), C=horizontal shift. Max=D+A, min=D−A.
- Wave combination: $a\sin x + b\cos x = R\sin(x+\varphi)$, where $R=\sqrt{(a^2+b^2)}$ and $\tan\varphi=b/a$.
- Solving strategy: use identities to get ONE trig ratio, factorise if quadratic-in-trig, solve, then list all solutions in the required interval.

Key skills

1. Convert 135° to radians (exact), and convert $5\pi/6$ radians to degrees.

2. Write down the exact value of $\cos(2\pi/3)$.

3. Use a compound-angle formula to find the exact value of $\sin(75°)$.

4. Simplify $1 - 2\sin^2\theta$ and simplify $2\sin\theta\cos\theta$.

Achieved

5. Solve $2\sin x - 1 = 0$ for $0 \le x \le 2\pi$.

6. Write the general solution (in radians) of $\cos\theta = -1/2$.

7. A and B are acute angles with $\sin A = 3/5$ and $\cos B = 5/13$. Find the exact value of $\sin(A + B)$.

8. A Ferris wheel at Ōrākei has radius 25 m and its centre is 30 m above the ground. It completes one revolution every 20 seconds, starting from the lowest point. Its height is $h(t) = 30 - 25\cos(\pi t/10)$ metres. State the amplitude and period, and find the height after 5 seconds.

Merit

9. Solve $2\cos^2 x + \cos x - 1 = 0$ for $0 \le x \le 2\pi$.

10. Solve $\sin 2x = \cos x$ for $0 \le x \le 2\pi$.

11. The depth of water at a Manukau Harbour jetty is modelled by $d = 8 + 3\sin(\pi t/6)$ metres, where t is hours after midnight. A boat needs a depth of at least 9.5 m to enter. During the first 12 hours ($0 \le t \le 12$), for what time interval is the depth at least 9.5 m?

Excellence

12. Prove the identity $(1 + \cos 2\theta)/\sin 2\theta = \cot\theta$, and state any values of θ for which it is undefined.

13. Express $3\sin x + 4\cos x$ in the form $R\sin(x + \varphi)$ with $R > 0$ and $0 < \varphi < \pi/2$, then find the general solution of $3\sin x + 4\cos x = 2.5$.

14. Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.