# NCEA Level 3 · AS 91578: Differentiation Toolkit

Name: _____ Date: _____

Hi! This standard is all about differentiating trickier functions (using product, quotient and chain rules, plus trig, e^x and ln) and then using those derivatives to solve real problems, rates, optimisation, tangents, curve shapes and motion. Work through the sheet top to bottom: warm up on the key skills, then build toward the Merit and Excellence reasoning.

**Revision summary**

- Rules: product $(uv)' = u'v + uv'$; quotient $(u/v)' = (u'v - uv')/v^2$; chain $dy/dx = dy/du \cdot du/dx$.
- Standard derivatives: $d/dx(\sin x)=\cos x$, $d/dx(\cos x)=-\sin x$, $d/dx(\tan x)=\sec^2 x$, $d/dx(e^x)=e^x$, $d/dx(\ln x)=1/x$.
- Chain versions: $d/dx(e^{(f)})=f' \cdot e^{(f)}$; $d/dx(\ln f)=f'/f$; $d/dx(\sin f)=f' \cdot \cos f$.
- Stationary points: solve $f'(x)=0$. Classify with sign of $f'(x)$ or with $f''(x)$: $f''>0$ minimum, $f''<0$ maximum, test if $f''=0$.
- Optimisation: write the quantity in one variable using a constraint, differentiate, set derivative = 0, then justify it is a max/min.
- Related rates: differentiate with respect to t; chain the rates together, e.g. $dV/dt = dV/dr \cdot dr/dt$.
- Kinematics: velocity $v = ds/dt$, acceleration $a = dv/dt = d^2s/dt^2$. Object at rest when $v = 0$.
- Tangent at $x = a$: gradient $m = f'(a)$, line $y - f(a) = m(x - a)$.

## Key skills

1. Differentiate $y = (3x^2 + 1)^5$.

2. Differentiate $y = x^2 e^x$.

3. Differentiate $y = (2x + 1)/(x - 3)$.

4. Differentiate $y = \ln(\sin x)$.

## Achieved

5. Find the gradient of the curve $y = 3\sin(2x)$ at $x = \pi/6$.

6. Find the coordinates of the stationary points of $y = x^3 - 6x^2 + 9x + 2$.

7. A trolley moves so its position is $s = t^3 - 4t^2 + 2t$ metres after t seconds. Find its velocity at $t = 3$ s.

8. Find the equation of the tangent to $y = e^{(2x)}$ at the point where $x = 0$.

## Merit

9. A farmer near Pukekohe uses 120 m of fencing to make a rectangular paddock against a straight river. No fence is needed along the river. Find the dimensions that give the maximum area, and state that maximum area.

10. A closed cylindrical can must hold 500 cm³. Find the radius that minimises the surface area (give it to 2 d.p.).

11. A spherical weather balloon is inflated so its radius increases at 2 cm/s. Find the rate at which its volume is increasing when the radius is 5 cm.

## Excellence

12. For the curve $y = x^3 - 3kx + 2$, determine all values of the constant $k$ for which the curve has two distinct stationary points. Justify your answer.

13. Using calculus, prove that among all rectangles with a fixed perimeter $P$, the square gives the greatest area.

14. A particle moves along a line with displacement $x(t) = e^{(-t)} \sin t$ metres for $t \geq 0$. Show that the particle is instantaneously at rest whenever $\tan t = 1$, and explain why the exponential factor never causes it to stop.