NCEA Level 3 · AS 91586: Working with Probability Distributions

Name: Date:

Hi! This standard is all about choosing the right probability model (normal, binomial, Poisson, uniform or triangular) and using it to answer real questions. Work through each section in order, check your working against the answers, and pay attention to when you have to justify your choice of model, that's where the top grades live.

Revision summary

- Normal: standardise with $z = (x - \mu)/\sigma$, then use tables/calculator for $P(Z < z)$. Inverse: given a probability, find z first, then $x = \mu + z\sigma$.
- Binomial $X \sim B(n, p)$: fixed n independent trials, constant p. $P(X = k) = C(n,k)\, p^k (1-p)^{(n-k)}$; mean = np, variance = np(1−p). Use when counting successes in a set number of trials.
- Poisson $X \sim Po(\lambda)$: events at a constant average rate over a fixed interval. $P(X = k) = e^{(-\lambda)} \lambda^k / k!$; mean = variance = λ. Scale λ in proportion to the interval length.
- Uniform (continuous on [a,b]): constant density $1/(b-a)$; $P(c < X < d) = (d-c)/(b-a)$; mean = (a+b)/2.
- Triangular(a, b, c) with min a, max b, mode c: peak height $2/(b-a)$. $P(X \le c) = (c-a)/(b-a)$. Areas are triangles, so use ½ × base × height.
- Model choice: binomial = fixed trials with success/failure; Poisson = counts over time/space with a rate (use as approx to binomial when n large, p small, λ = np); normal = continuous, symmetric, bell-shaped data.
- Useful z-values: $P(Z<z)=0.90 \rightarrow z=1.2816$; $0.95 \rightarrow 1.6449$; $0.975 \rightarrow 1.96$; 0.02 lower tail → z=−2.054.
- Normal approximation to binomial ($np \ge 5$ and $n(1-p) \ge 5$): $\mu = np$, $\sigma = \sqrt{(np(1-p))}$, and apply a continuity correction of ±0.5.

Key skills

1. X is normally distributed with mean 50 and standard deviation 8. Find $P(X < 60)$.

2. $X \sim$ Binomial($n = 10$, $p = 0.3$). Find $P(X = 4)$.

3. The number of texts Aroha gets per hour follows a Poisson distribution with mean 3. Find $P(X = 2)$.

4. A bus arrival delay is uniformly distributed on the interval [0, 20] minutes. Find $P(5 < X < 12)$.

Achieved

5. The weight of a kererū is normally distributed with mean 650 g and standard deviation 40 g. Find the probability a randomly chosen kererū weighs more than 700 g.

6. A 12-question multiple-choice quiz has four options per question. Manu guesses every answer, so the chance each is correct is 0.25. Find the probability he gets exactly 3 correct.

7. A café receives online orders at an average rate of 5 per hour, following a Poisson distribution. Find the probability that fewer than 3 orders arrive in a given hour.

8. Delivery times for a courier follow a triangular distribution with minimum 20 min, mode 30 min and maximum 50 min. Find the probability a delivery takes less than 30 minutes.

Merit

9. A machine fills juice bottles so the volume is normally distributed with standard deviation 5 mL. The manager wants only 2% of bottles to fall below the labelled 500 mL. What mean fill volume should be set?

10. A factory line produces components that are defective 1.5% of the time, independently. A batch of 200 is checked. Explain why a Poisson model is suitable and find the probability that at most 2 components in the batch are defective.

11. Bags of chips have weights that are normally distributed with mean 150 g and standard deviation 4 g. In a shipment of 500 bags, how many would you expect to weigh between 145 g and 155 g?

Excellence

12. A test score X is normally distributed. It is known that 10% of students score below 40 and 5% score above 75. Find the mean and standard deviation of the distribution.

13. Minor earthquakes felt in Auckland occur at an average rate of 0.8 per year, modelled as Poisson. Find the probability that at least 5 are felt over a 5–year period, and state one assumption the model relies on.

14. A fair coin is tossed 400 times. Using an appropriate distribution, estimate the probability of getting at least 210 heads, and justify why your chosen method is valid.