NCEA Level 3 · AS 91587: Systems of Simultaneous Equations

Name: _____ Date: _____

This standard is all about solving and modelling problems with three unknowns using Gaussian elimination, and knowing when a system has one solution, infinitely many, or none. Work through the sections in order, the Key skills warm up your method, then push into the Merit and Excellence reasoning that earns the top grades.

Revision summary

- A system of 3 equations in 3 unknowns (x, y, z) usually represents three planes; the solution is where they all meet.
- Gaussian elimination: use row operations (swap rows, multiply a row by a non-zero number, add a multiple of one row to another) to reach upper-triangular form, then back-substitute.
- Unique solution: elimination gives a leading variable in every row (three planes meet at one point).
- Infinite solutions: a row becomes 0 = 0 (a redundant equation); write the answer parametrically, e.g. let z = t and express x and y in terms of t.
- No solution: a row becomes 0 = k where k ≠ 0 (a contradiction, the planes have no common point).
- To model: define each unknown clearly, turn each condition into one linear equation, then solve and interpret the answer in context.
- For a system with a parameter, isolate the parameter in the final row: (parameter)·variable = value decides unique / infinite / none.
- Always check a solution by substituting back into every original equation.

Key skills

1. Solve this triangular system by back-substitution: $x + 2y - z = 3$; $y + 2z = 5$; $z = 2$.

2. Use elimination to solve: $2x + y - z = 3$; $x - y + 2z = 1$; $x + y + z = 4$.

3. A system reduces so that its final row becomes: (a) $0 = 0$, (b) $0 = 7$, (c) a leading variable in each of the three rows. State the number of solutions in each case.

4. Write the solution set in parametric form: $x + y + z = 5$; $x + 2y + 3z = 8$; $2x + 3y + 4z = 13$.

Achieved

5. Solve the system: $x + y + z = 100$; $2x + y + 3z = 180$; $x + 2y + z = 150$.

6. At a Avondale café, three orders are placed. Order 1: 2 pies, 3 coffees and 1 muffin cost $26. Order 2: 1 pie, 2 coffees and 2 muffins cost $20. Order 3: 3 pies, 1 coffee and 1 muffin cost $20. Find the price of each item.

7. Determine whether the system has a unique solution, infinitely many, or none: $x + y + z = 4$; $2x + 2y + 2z = 8$; $x - y + z = 2$. If infinite, give the solution parametrically.

8. $10 000 is split between three accounts earning 3%, 4% and 5% per year. The total interest for the year is $370, and the amount in the first account equals the total of the other two. How much is in each account?

Merit

9. A concert sells adult, student and child tickets at $25, $15 and $10. A total of 500 tickets are sold for total revenue of $9000, and the number of students is twice the number of children. Find how many of each ticket were sold.

10. A parabola $y = ax^2 + bx + c$ passes through the points (1, 6), (2, 11) and (3, 18). Find its equation, connecting the points to a system of equations.

11. Reduce and interpret: $x + 2y - z = 3$; $2x - y + 3z = 4$; $x - 3y + 4z = 1$. State the solution type and give the general solution.

Excellence

12. For the system $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + kz = m$, find the value(s) of k and m for which the system has (i) a unique solution, (ii) infinitely many solutions, (iii) no solution. Justify your answer.

13. Investigate the system $ax + y + z = 1$; $x + ay + z = 1$; $x + y + az = 1$. Determine all values of a giving a unique solution, infinitely many, or none, and justify.

14. Balancing the combustion reaction $a\cdot C_3H_8 + b\cdot O_2 \rightarrow c\cdot CO_2 + d\cdot H_2O$ leads to a system for a, b, c, d. Set up the equations, show it has infinitely many solutions, and explain how a valid balanced equation is chosen.