NCEA Level 1 · AS 91947: Mathematical Reasoning (Number & Algebra)

Name: _____ Date: _____

Kia ora! This standard is all about showing clear algebraic reasoning, solving equations, working with expressions, and generalising patterns. Work through each section in order, always writing out your steps, because in the exam your reasoning earns the marks, not just the final answer.

Revision summary

- Expanding: $a(b + c) = ab + ac$; $(x + p)(x + q) = x^2 + (p + q)x + pq$.
- Factorising: common factor first, then quadratics $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$. Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$.
- Solving linear equations: do the same operation to both sides to isolate the variable. For quadratics, factorise and use the null factor law (if $AB = 0$ then $A = 0$ or $B = 0$).
- Simultaneous equations: use substitution or elimination to find the point that satisfies both equations.
- Exponent laws: $x^a \times x^b = x^{a+b}$; $x^a \div x^b = x^{a-b}$; $(x^a)^b = x^{ab}$; $x^0 = 1$; $x^{-a} = 1/x^a$.
- Inequalities: solve like equations, but reverse the sign (< becomes >) when multiplying or dividing by a negative.
- Proportional reasoning: set up equivalent ratios or a rate, then scale up or down.
- Generalising a linear pattern: term = (common difference) × n + (constant); always justify why the rule works from the structure of the pattern.

Key skills

1. Expand and simplify: $3(2x - 5) + 4x$.

2. Factorise fully: $x^2 + 7x + 12$.

3. Solve for x: $5x - 8 = 27$.

4. Simplify: $(2a^3)^2 \times a^4$.

Achieved

5. Aroha buys 3 iceblocks and a $2 juice at the Avondale dairy and pays $9.50. Write and solve an equation to find the price of one iceblock.

6. Solve the quadratic equation $x^2 - 5x - 14 = 0$.

7. Solve the inequality $4x + 3 \leq 19$ and describe the solution.

8. A recipe for 4 people needs 600 g of kūmara. How much kūmara is needed for 10 people?

Merit

9. A rectangle has length 3 cm more than its width. Its area is 40 cm². Set up and solve an equation to find the dimensions.

10. Solve the simultaneous equations $2x + y = 11$ and $3x - y = 4$, and explain what the solution represents.

11. Simplify the expression $(x^2 - 9) \div (x + 3)$, showing your reasoning.

Excellence

12. A pattern of square tiles grows so that stage 1 uses 4 tiles, stage 2 uses 7 tiles, stage 3 uses 10 tiles. Find a rule for the number of tiles at stage n, justify why it works, and determine which stage uses 100 tiles.

13. Prove that the sum of any three consecutive whole numbers is always a multiple of 3.

14. For what values of k does the equation $x^2 + kx + 9 = 0$ have exactly one solution? Justify your answer.

Answer key (for parents and tutors)

1. Expand and simplify: $3(2x - 5) + 4x$.
   $3(2x - 5) = 6x - 15$. Adding $4x$ gives $6x - 15 + 4x = 10x - 15$.
   Achieved fluency: applies the distributive law and collects like terms.

2. Factorise fully: $x^2 + 7x + 12$.
   Find two numbers with product 12 and sum 7: these are 3 and 4. So $x^2 + 7x + 12 = (x + 3)(x + 4)$.
   Achieved fluency: factorises a monic quadratic.

3. Solve for x: $5x - 8 = 27$.
   Add 8 to both sides: $5x = 35$. Divide by 5: $x = 7$.
   Achieved fluency: solves a two-step linear equation.

4. Simplify: $(2a^3)^2 \times a^4$.
   $(2a^3)^2 = 4a^6$. Then $4a^6 \times a^4 = 4a^{10}$ (add the exponents).
   Achieved fluency: applies exponent laws with a coefficient.

5. Aroha buys 3 iceblocks and a $2 juice at the Avondale dairy and pays $9.50. Write and solve an equation to find the price of one iceblock.
   Let the iceblock price be x. Then $3x + 2 = 9.50$. Subtract 2: $3x = 7.50$. Divide by 3: $x = 2.50$. Each iceblock costs $2.50.
   Achieved: forms and solves a linear equation from a word context.

6. Solve the quadratic equation $x^2 - 5x - 14 = 0$.
   Two numbers with product -14 and sum -5 are -7 and 2. So $(x - 7)(x + 2) = 0$. By the null factor law, $x = 7$ or $x = -2$.
   Achieved: factorises and solves a quadratic equation.

7. Solve the inequality $4x + 3 \leq 19$ and describe the solution.
   Subtract 3: $4x \leq 16$. Divide by 4: $x \leq 4$. The solution is all values of x less than or equal to 4.
   Achieved: solves a linear inequality.

8. A recipe for 4 people needs 600 g of kūmara. How much kūmara is needed for 10 people?
   Kūmara per person = $600 \div 4 = 150$ g. For 10 people: $150 \times 10 = 1500$ g (1.5 kg).
   Achieved: uses proportional reasoning via a unit rate.

9. A rectangle has length 3 cm more than its width. Its area is 40 cm². Set up and solve an equation to find the dimensions.
   Let the width be w. Then length = $w + 3$ and area = $w(w + 3) = 40$. Expand: $w^2 + 3w = 40$, so $w^2 + 3w - 40 = 0$. Factorise: two numbers with product -40 and sum 3 are 8 and -5, giving $(w + 8)(w - 5) = 0$. So $w = -8$ or $w = 5$. Width cannot be negative, so $w = 5$ cm and length = 8 cm.
   Merit: links a geometric context to a quadratic and rejects the invalid solution.

10. Solve the simultaneous equations $2x + y = 11$ and $3x - y = 4$, and explain what the solution represents.
    Add the two equations to eliminate y: $(2x + y) + (3x - y) = 11 + 4$, so $5x = 15$, $x = 3$. Substitute into the first equation: $2(3) + y = 11$, so $y = 5$. The solution (3, 5) is the point where the two straight lines cross.
    Merit: solves simultaneously and connects the answer to a graphical meaning.

11. Simplify the expression $(x^2 - 9) \div (x + 3)$, showing your reasoning.
    $x^2 - 9$ is a difference of two squares: $x^2 - 9 = (x + 3)(x - 3)$. So $(x + 3)(x - 3) \div (x + 3) = x - 3$, provided $x \neq -3$ (so we are not dividing by zero).
    Merit: connects difference-of-squares factorising with cancelling and notes the restriction.

12. A pattern of square tiles grows so that stage 1 uses 4 tiles, stage 2 uses 7 tiles, stage 3 uses 10 tiles. Find a rule for the number of tiles at stage n, justify why it works, and determine which stage uses 100 tiles.
    The numbers 4, 7, 10 increase by 3 each stage, so the rule is linear: tiles = $3n + c$. At $n = 1$, $3(1) + c = 4$, so $c = 1$. Rule: tiles = $3n + 1$. This works because each new stage adds a fixed group of 3 tiles to a starting core of 1 tile. Setting $3n + 1 = 100$ gives $3n = 99$, $n = 33$. Stage 33 uses 100 tiles.
    Excellence: generalises a pattern, justifies the rule from structure, and solves in reverse.

13. Prove that the sum of any three consecutive whole numbers is always a multiple of 3.
    Let the three consecutive numbers be n, n + 1 and n + 2. Their sum is n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1). Since 3(n + 1) has a factor of 3, the sum is always a multiple of 3 for any whole number n.
    Excellence: constructs a general algebraic proof of a number property.

14. For what values of k does the equation $x^2 + kx + 9 = 0$ have exactly one solution? Justify your answer.
    A quadratic has exactly one solution when it is a perfect square, i.e. when the discriminant $b^2 - 4ac = 0$. Here a = 1, b = k, c = 9, so $k^2 - 4(1)(9) = 0$, giving $k^2 = 36$, so k = 6 or k = −6. Check: for k = 6, $x^2 + 6x + 9 = (x + 3)^2 = 0$ gives the single repeated root x = −3; for k = −6, $(x - 3)^2 = 0$ gives x = 3. So k = 6 or k = −6.
    Excellence: reasons abstractly about repeated roots using the discriminant and verifies.