Year 10 · Algebra: Equations, Lines & Quadratics

Name: _____ Date: _____

Kia ora! This sheet pulls together the algebra you've been learning: tidying up expressions, solving equations and inequalities, working with straight lines, and getting started on quadratics. Take your time and show every step.

Warm-up

1. Collect like terms: $5a + 3b - 2a + 7b$.

2. Expand: $4(2x - 5)$.

3. Solve: $3x + 7 = 22$.

4. Factorise using a common factor: $6y + 15$.

Practice

5. Expand and simplify: $(x + 4)(x - 3)$.

6. Factorise the quadratic: $x^2 + 7x + 12$.

7. Solve the inequality and show the solution on a number line: $-2x + 5 \leq 11$.

8. Mia hires a kayak at Lake Pupuke. It costs a $12 booking fee plus $8 per hour. If she pays $52 in total, form and solve an equation to find how many hours she hired the kayak.

9. A water tank at a farm near Kumeū starts with 20 litres and is filled at a steady 15 litres per minute. Write the equation for the volume V (litres) after t minutes in the form $V = mt + c$, then find the volume after 6 minutes.

10. Solve the quadratic equation $(x - 2)(x + 5) = 0$, and state where the related graph $y = (x - 2)(x + 5)$ crosses the x-axis.

Challenge

11. A straight line passes through the points $(1, 5)$ and $(4, 14)$. Find its gradient, then find the equation of the line in the form $y = mx + c$.

12. Factorise $x^2 + 2x - 8$ by grouping, then describe how the graph of $y = x^2 + 3$ differs from the graph of $y = x^2$.

Answer key (for parents and tutors)

1. Collect like terms: $5a + 3b - 2a + 7b$.
   Group the a-terms and the b-terms: $(5a - 2a) + (3b + 7b) = 3a + 10b$.

2. Expand: $4(2x - 5)$.
   Multiply each term inside the bracket by 4: $4 \times 2x = 8x$ and $4 \times (-5) = -20$. Answer: $8x - 20$.

3. Solve: $3x + 7 = 22$.
   Subtract 7 from both sides: $3x = 15$. Divide both sides by 3: $x = 5$.

4. Factorise using a common factor: $6y + 15$.
   The highest common factor of 6 and 15 is 3. $6y + 15 = 3(2y + 5)$.

5. Expand and simplify: $(x + 4)(x - 3)$.
   Use each term of the first bracket times each term of the second: $x \times x = x^2$, $x \times (-3) = -3x$, $4 \times x = 4x$, $4 \times (-3) = -12$. Combine: $x^2 - 3x + 4x - 12 = x^2 + x - 12$.

6. Factorise the quadratic: $x^2 + 7x + 12$.
   Find two numbers that multiply to 12 and add to 7. These are 3 and 4 ($3 \times 4 = 12$, $3 + 4 = 7$). So $x^2 + 7x + 12 = (x + 3)(x + 4)$.

7. Solve the inequality and show the solution on a number line: $-2x + 5 \leq 11$.
   Subtract 5 from both sides: $-2x \leq 6$. Divide both sides by -2 and reverse the inequality (dividing by a negative): $x \geq -3$. On a number line, draw a filled (closed) circle at -3 and shade the arrow to the right.

8. Mia hires a kayak at Lake Pupuke. It costs a $12 booking fee plus $8 per hour. If she pays $52 in total, form and solve an equation to find how many hours she hired the kayak.
   Let h = number of hours. The cost equation is $8h + 12 = 52$. Subtract 12: $8h = 40$. Divide by 8: $h = 5$. Mia hired the kayak for 5 hours.

9. A water tank at a farm near Kumeū starts with 20 litres and is filled at a steady 15 litres per minute. Write the equation for the volume V (litres) after t minutes in the form $V = mt + c$, then find the volume after 6 minutes.
   The starting amount is the y-intercept, $c = 20$. The rate of change is the gradient, $m = 15$. So $V = 15t + 20$. After 6 minutes: $V = 15 \times 6 + 20 = 90 + 20 = 110$ litres.

10. Solve the quadratic equation $(x - 2)(x + 5) = 0$, and state where the related graph $y = (x - 2)(x + 5)$ crosses the x-axis.
    By the zero product property, at least one bracket must equal 0. Either $x - 2 = 0$, giving $x = 2$, or $x + 5 = 0$, giving $x = -5$. The graph crosses the x-axis (the x-intercepts) at $(2, 0)$ and $(-5, 0)$.

11. A straight line passes through the points $(1, 5)$ and $(4, 14)$. Find its gradient, then find the equation of the line in the form $y = mx + c$.
    Gradient $m = \Delta y \div \Delta x = (14 - 5) \div (4 - 1) = 9 \div 3 = 3$. Substitute $m = 3$ and the point $(1, 5)$ into $y = mx + c$: $5 = 3 \times 1 + c$, so $5 = 3 + c$, giving $c = 2$. The equation is $y = 3x + 2$. (Check with $(4, 14)$: $3 \times 4 + 2 = 14$ ✓.)

12. Factorise $x^2 + 2x - 8$ by grouping, then describe how the graph of $y = x^2 + 3$ differs from the graph of $y = x^2$.
    Factorising by grouping: split the middle term using two numbers that multiply to -8 and add to 2, which are 4 and -2. $x^2 + 2x - 8 = x^2 + 4x - 2x - 8 = x(x + 4) - 2(x + 4) = (x - 2)(x + 4)$. For the graphs: $y = x^2$ has its lowest point (vertex) at $(0, 0)$. Adding 3 to give $y = x^2 + 3$ shifts the whole parabola up by 3 units, so its vertex is at $(0, 3)$. The shape and width stay exactly the same; only the vertical position changes.