Year 10 · Number: Powers, Roots, Ratios & Rates

Name: _____ Date: _____

Kia ora! This sheet mixes exponent rules, roots, scientific notation, percentages, ratios and rates. Show your working and keep exact answers exact where you can, no calculator needed for most questions.

Warm-up

1. Use an exponent rule to simplify and then evaluate $2^3 \times 2^4$.

2. Simplify $\sqrt{3} + \sqrt{3}$.

3. Write 0.75 as a fraction in simplest form and as a percentage.

4. Evaluate $(-2)^3$.

Practice

5. From this list, write down which numbers are irrational: $\sqrt{16}$, $\sqrt{10}$, π, $\sqrt[3]{27}$, 0.6.

6. Evaluate 3^{-2} and 5^0.

7. Write 45 600 in scientific notation to 3 significant figures.

8. Simplify $\sqrt{2} \times \sqrt{8}$, then simplify $\sqrt{50} - \sqrt{32}$.

9. At Pak'nSave in Avondale, a block of cheese went up from $12.00 to $15.00. What is the percentage increase?

10. Aria drives from Avondale to Hamilton, a distance of 120 km, in 1½ hours. What is her average speed in km/h? If 30% of the trip is on the motorway, how many kilometres is that?

11. A jacket is on sale for $68 after a 15% discount. What was the original price?

Challenge

12. Evaluate exactly, showing order of operations: $\sqrt{(3^2 + 4^2)} + (-2)^3 \times 2$.

13. Light travels at about 3.0×10^8 metres per second. The Sun is about 1.5×10^{11} metres from Earth. How long (in seconds) does sunlight take to reach Earth? Give your answer to 2 significant figures.

Answer key (for parents and tutors)

1. Use an exponent rule to simplify and then evaluate $2^3 \times 2^4$.
   Product-of-exponents rule: $a^m \times a^n = a^{m+n}$, so $2^3 \times 2^4 = 2^{3+4} = 2^7$. Then $2^7 = 128$.

2. Simplify $\sqrt{3} + \sqrt{3}$.
   Only like roots add. $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$ (two lots of $\sqrt{3}$).

3. Write 0.75 as a fraction in simplest form and as a percentage.
   0.75 = 75/100 = 3/4 (dividing top and bottom by 25). As a percentage, 0.75 = 75%.

4. Evaluate $(-2)^3$.
   $(-2)^3 = (-2) \times (-2) \times (-2)$. First $(-2) \times (-2) = 4$, then $4 \times (-2) = -8$. So $(-2)^3 = -8$.

5. From this list, write down which numbers are irrational: $\sqrt{16}$, $\sqrt{10}$, π, $\sqrt[3]{27}$, 0.6.
   $\sqrt{16} = 4$ (rational). $\sqrt{10}$ is not a perfect square, so it is a non-repeating infinite decimal, irrational. π is irrational. $\sqrt[3]{27} = 3$ (rational). 0.6 = 3/5 (rational). Irrational numbers: $\sqrt{10}$ and π. (General rule: a root of a whole number is rational only if the number is a perfect square/cube.)

6. Evaluate 3^{-2} and 5^0.
   Negative exponent rule: $a^{-m} = 1/a^m$, so $3^{-2} = 1/3^2 = 1/9$. Zero exponent rule: $a^0 = 1$ ($a \neq 0$), so $5^0 = 1$.

7. Write 45 600 in scientific notation to 3 significant figures.
   Move the decimal point so the first factor is between 1 and 10: 4.56×10^4 (the point moved 4 places left). The value 4.56 already has 3 significant figures, so $45\,600 = 4.56 \times 10^4$.

8. Simplify $\sqrt{2} \times \sqrt{8}$, then simplify $\sqrt{50} - \sqrt{32}$.
   $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$: $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$. For the second, write each as a multiple of $\sqrt{2}$: $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$ and $\sqrt{32} = \sqrt{(16 \times 2)} = 4\sqrt{2}$. So $\sqrt{50} - \sqrt{32} = 5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$.

9. At Pak'nSave in Avondale, a block of cheese went up from $12.00 to $15.00. What is the percentage increase?
   Increase = $15.00 − $12.00 = $3.00. Percentage increase = increase ÷ original × 100 = 3 ÷ 12 × 100 = 0.25 × 100 = 25%. The price rose by 25%.

10. Aria drives from Avondale to Hamilton, a distance of 120 km, in 1½ hours. What is her average speed in km/h? If 30% of the trip is on the motorway, how many kilometres is that?
    Speed = distance ÷ time = 120 ÷ 1.5 = 80 km/h. Motorway distance = 30% of 120 km = 0.30 × 120 = 36 km.

11. A jacket is on sale for $68 after a 15% discount. What was the original price?
    After a 15% decrease, $68 is 85% of the original (100% − 15% = 85%). So 0.85 × original = 68. Original = 68 ÷ 0.85 = $80.00. Check: 15% of 80 = $12, and 80 − 12 = $68. ✓

12. Evaluate exactly, showing order of operations: $\sqrt{(3^2 + 4^2)} + (-2)^3 \times 2$.
    Use GEMA. Grouped first: inside the root, $3^2 + 4^2 = 9 + 16 = 25$, so $\sqrt{25} = 5$. Exponent: $(-2)^3 = -8$. Multiplicative: $-8 \times 2 = -16$. Additive: $5 + (-16) = -11$. So the answer is −11.

13. Light travels at about 3.0×10^8 metres per second. The Sun is about 1.5×10^{11} metres from Earth. How long (in seconds) does sunlight take to reach Earth? Give your answer to 2 significant figures.
    Time = distance ÷ speed = $(1.5 \times 10^{11}) \div (3.0 \times 10^8)$. Divide the number parts: $1.5 \div 3.0 = 0.5$. Divide the powers using the quotient rule: $10^{11} \div 10^8 = 10^{11-8} = 10^3$. So time = $0.5 \times 10^3 = 500$ seconds. To 2 significant figures, that is 5.0×10^2 seconds (about 500 s, or 8 minutes 20 seconds).