Year 10 · Probability: Chance in Action: Experiment vs Theory

Name: _____ Date: _____

Hi! This sheet helps you see how the probabilities we predict on paper compare to what actually happens when we run experiments. Grab a die or a coin if you can, and keep an eye on how bigger experiments give steadier results.

Warm-up

1. A fair six-sided die is rolled once. What is the theoretical probability of rolling a number greater than 4? Give your answer as a fraction in simplest form.

2. A spinner was spun 50 times and landed on red 20 times. What is the experimental probability (relative frequency) of landing on red? Give a fraction, a decimal and a percentage.

3. A bag holds 3 red, 5 blue and 2 green marbles. One marble is drawn at random. What is P(blue)?

4. Using the same bag as the question above, what is P(not blue)?

Practice

5. Two fair coins are tossed together. (a) List all the outcomes in the sample space. (b) Find P(two heads). (c) Find P(at least one head).

6. Aria finds an old drawing pin and wonders about the chance it lands point-up. She tosses it 40 times and it lands point-up 24 times. (a) Why can Aria only find this probability by experiment, not by calculation? (b) What is her experimental probability of the pin landing point-up?

7. Tane rolled a fair die 120 times and recorded: 1 → 18, 2 → 25, 3 → 19, 4 → 22, 5 → 16, 6 → 20. (a) What is the theoretical probability of each number, and roughly how many of each would you expect in 120 rolls? (b) Compare the experimental probability of rolling a 2 with its theoretical probability. (c) Explain why the results are not exactly equal to the expected values.

8. At a Vertex Academy games night, two fair dice are rolled and the two numbers are added. (a) Systematically list or tabulate the sample space of possible totals and count the outcomes. (b) Find P(total = 8). (c) Find P(total is 10 or more).

9. A four-colour spinner (red, blue, yellow, green) is spun 80 times with these results: red 26, blue 18, yellow 14, green 22. (a) Calculate the experimental probability of each colour as a decimal. (b) Describe how you would draw a data visualisation of these results and what it would show.

10. On the way to Avondale School, Mia passes a set of traffic lights that is red 40% of the time, and independently her bus is late 15% of the time. (a) Explain why these two events are independent. (b) Find the probability that the light is red AND the bus is late. (c) Explain why you cannot simply add 0.40 and 0.15 to find P(red or late).

Challenge

11. Leo uses a spreadsheet to simulate tossing a fair coin. In 30 tosses he gets 12 heads; in 100 tosses he gets 46 heads; in 1000 tosses he gets 508 heads. (a) Find the experimental probability of heads for each set of trials. (b) The theoretical probability is 0.5. Which result is closest, and why? (c) State the Law of Large Numbers and explain what it predicts if Leo ran 100 000 tosses.

12. A bag contains 4 red and 6 blue marbles. Two marbles are drawn one after the other WITHOUT replacement. (a) Draw a tree diagram (describe the branches and their probabilities). (b) Find P(both red). (c) Find P(one of each colour).

Answer key (for parents and tutors)

1. A fair six-sided die is rolled once. What is the theoretical probability of rolling a number greater than 4? Give your answer as a fraction in simplest form.
   Numbers greater than 4 are 5 and 6, so 2 favourable outcomes out of 6 possible. P(more than 4) = 2/6 = 1/3.

2. A spinner was spun 50 times and landed on red 20 times. What is the experimental probability (relative frequency) of landing on red? Give a fraction, a decimal and a percentage.
   Experimental probability = number of reds ÷ total trials = 20/50. Simplify: 20/50 = 2/5 = 0.4 = 40%.

3. A bag holds 3 red, 5 blue and 2 green marbles. One marble is drawn at random. What is P(blue)?
   Total marbles = 3 + 5 + 2 = 10. Blue marbles = 5. P(blue) = 5/10 = 1/2.

4. Using the same bag as the question above, what is P(not blue)?
   P(not blue) = 1 − P(blue) = 1 − 1/2 = 1/2. (Check: 3 red + 2 green = 5 non-blue out of 10 = 5/10 = 1/2.)

5. Two fair coins are tossed together. (a) List all the outcomes in the sample space. (b) Find P(two heads). (c) Find P(at least one head).
   (a) Sample space = {HH, HT, TH, TT}, 4 equally likely outcomes. (b) Only HH gives two heads, so P(two heads) = 1/4. (c) Outcomes with at least one head are HH, HT, TH = 3 outcomes, so P(at least one head) = 3/4.

6. Aria finds an old drawing pin and wonders about the chance it lands point-up. She tosses it 40 times and it lands point-up 24 times. (a) Why can Aria only find this probability by experiment, not by calculation? (b) What is her experimental probability of the pin landing point-up?
   (a) A drawing pin is not a regular, symmetrical shape, so the two ways it can land (point-up or point-down) are not equally likely. There is no way to work out the theoretical probability just by counting equal outcomes, so a physical experiment is the only way to explore it. (b) Experimental P(point-up) = 24/40 = 3/5 = 0.6 = 60%.

7. Tane rolled a fair die 120 times and recorded: 1 → 18, 2 → 25, 3 → 19, 4 → 22, 5 → 16, 6 → 20. (a) What is the theoretical probability of each number, and roughly how many of each would you expect in 120 rolls? (b) Compare the experimental probability of rolling a 2 with its theoretical probability. (c) Explain why the results are not exactly equal to the expected values.
   (a) Each face is equally likely, so theoretical probability = 1/6 for every number. Expected count = 120 × 1/6 = 20 of each. (b) Experimental P(2) = 25/120 ≈ 0.208, while theoretical P(2) = 1/6 ≈ 0.167. The 2 came up a bit more often than expected. (c) Because each roll is random, results from repeated trials naturally vary around the expected values, you rarely get exactly 20 of each in 120 rolls. This variation is normal chance behaviour.

8. At a Vertex Academy games night, two fair dice are rolled and the two numbers are added. (a) Systematically list or tabulate the sample space of possible totals and count the outcomes. (b) Find P(total = 8). (c) Find P(total is 10 or more).
   (a) There are 6 × 6 = 36 equally likely outcomes (first die, second die). The totals table shows: total 2 (1 way), 3 (2), 4 (3), 5 (4), 6 (5), 7 (6), 8 (5), 9 (4), 10 (3), 11 (2), 12 (1). These add to 36. (b) Total 8 happens 5 ways (2+6, 3+5, 4+4, 5+3, 6+2), so P(8) = 5/36. (c) Totals of 10, 11 or 12 have 3 + 2 + 1 = 6 ways, so P(10 or more) = 6/36 = 1/6.

9. A four-colour spinner (red, blue, yellow, green) is spun 80 times with these results: red 26, blue 18, yellow 14, green 22. (a) Calculate the experimental probability of each colour as a decimal. (b) Describe how you would draw a data visualisation of these results and what it would show.
   (a) P(red) = 26/80 = 0.325; P(blue) = 18/80 = 0.225; P(yellow) = 14/80 = 0.175; P(green) = 22/80 = 0.275. (These add to 1.0, a good check.) (b) A bar graph works well: put the four colours along the horizontal axis and either frequency (or relative frequency) up the vertical axis, drawing a bar of the matching height for each colour. The graph would show red as the tallest bar and yellow the shortest, describing the distribution of observed outcomes, the spinner appears roughly even but red came up most and yellow least in this experiment.

10. On the way to Avondale School, Mia passes a set of traffic lights that is red 40% of the time, and independently her bus is late 15% of the time. (a) Explain why these two events are independent. (b) Find the probability that the light is red AND the bus is late. (c) Explain why you cannot simply add 0.40 and 0.15 to find P(red or late).
    (a) The colour of the traffic light has no effect on whether the bus is late (they have unrelated causes), so knowing one outcome does not change the probability of the other, that is what independent means. (b) For independent

events, multiply: P(red and late) = 0.40 × 0.15 = 0.06 = 6%. (c) Adding would double-count the outcomes where the light is red AND the bus is late. The correct rule is P(red or late) = 0.40 + 0.15 − 0.06 = 0.49, subtracting the overlap so it is only counted once.

11. Leo uses a spreadsheet to simulate tossing a fair coin. In 30 tosses he gets 12 heads; in 100 tosses he gets 46 heads; in 1000 tosses he gets 508 heads. (a) Find the experimental probability of heads for each set of trials. (b) The theoretical probability is 0.5. Which result is closest, and why? (c) State the Law of Large Numbers and explain what it predicts if Leo ran 100 000 tosses.
(a) 30 tosses: 12/30 = 0.40. 100 tosses: 46/100 = 0.46. 1000 tosses: 508/1000 = 0.508. (b) The 1000-toss result (0.508) is closest to 0.5. With more trials, one-off lucky or unlucky runs have less effect on the overall relative frequency, so the estimate settles nearer the true value. (c) The Law of Large Numbers states that as the number of trials increases, the experimental probability gets closer and closer to the theoretical probability. So with 100 000 tosses Leo's proportion of heads would almost certainly be extremely close to 0.5, much closer than any of his smaller experiments.

12. A bag contains 4 red and 6 blue marbles. Two marbles are drawn one after the other WITHOUT replacement. (a) Draw a tree diagram (describe the branches and their probabilities). (b) Find P(both red). (c) Find P(one of each colour).
(a) First draw branches: red with probability 4/10 and blue with probability 6/10. If red was drawn first, 9 marbles remain (3 red, 6 blue), so second-draw branches are red 3/9 and blue 6/9. If blue was drawn first, 9 remain (4 red, 5 blue), so second-draw branches are red 4/9 and blue 5/9. The probabilities change on the second draw because the events are dependent (no replacement). (b) P(both red) = 4/10 × 3/9 = 12/90 = 2/15. (c) P(one of each) = P(red then blue) + P(blue then red) = (4/10 × 6/9) + (6/10 × 4/9) = 24/90 + 24/90 = 48/90 = 8/15.