Year 13 Science: Circular motion and gravity

Name: _____ Date: _____

Try these without your notes first, then check the answer key.

Practice

1. A 0.050 kg ball on a string moves in a horizontal circle of radius 0.40 m, taking 0.50 s per revolution. Find the tension, ignoring gravity.

2. A bend of radius 120 m is banked at 12.0°. At what speed is no friction needed?

3. A roller coaster loop has radius 12.0 m at the top. What is the slowest the car can go at the top and still follow the track?

4. An 80.0 kg astronaut weighs 785 N on the Earth's surface. What is the gravitational force on her one Earth radius above the surface?

Exam-style question: an Earth observation satellite

A satellite that images New Zealand's forests orbits 700 km above the Earth's surface in a circular orbit. Earth's mass is 5.97×10^{24} kg, its radius 6.37×10^{6} m, and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

5. Show that the gravitational field strength at the satellite's height is 7.97 N kg^{-1}.

6. Calculate the satellite's orbital speed.

7. Calculate its orbital period in minutes.

8. Explain why the satellite keeps a constant speed without using its engines, even though a force acts on it the whole time.

Fix the mistake

9. Correct this statement: "The free-body diagram shows weight, tension and a centripetal force."

10. Correct this statement: "For a satellite 700 km up, $F = GMm \div (7.00 \times 10^{5})^2$."

11. Correct this statement: "Astronauts in orbit are weightless because there is no gravity up there."

12. Correct this statement: "A heavier satellite must go faster to stay in the same orbit."

Answer key (for parents and tutors)

1. A 0.050 kg ball on a string moves in a horizontal circle of radius 0.40 m, taking 0.50 s per revolution. Find the tension, ignoring gravity.
   $v = 2\pi r \div T = 2\pi \times 0.40 \div 0.50 = 5.03\ m\ s^{-1}$; $T = mv^2 \div r = 0.050 \times 5.03^2 \div 0.40 = 3.16\ N$.

2. A bend of radius 120 m is banked at 12.0°. At what speed is no friction needed?
   $\tan\theta = v^2 \div (rg)$, so $v = \sqrt{(rg \tan\theta)} = \sqrt{(120 \times 9.81 \times \tan 12.0°)} = \sqrt{250.2} = 15.8\ m\ s^{-1}$.

3. A roller coaster loop has radius 12.0 m at the top. What is the slowest the car can go at the top and still follow the track?
   At the slowest speed the track pushes with zero force and weight alone supplies the centripetal force: $v = \sqrt{(gr)} = \sqrt{(9.81 \times 12.0)} = 10.8\ m\ s^{-1}$.

4. An 80.0 kg astronaut weighs 785 N on the Earth's surface. What is the gravitational force on her one Earth radius above the surface?
   One Earth radius above the surface she is twice as far from the centre, so the force is a quarter: $785 \div 4 = 196\ N$.

5. Show that the gravitational field strength at the satellite's height is 7.97 N kg^{-1}.
   $r = 6.37 \times 10^6 + 7.00 \times 10^5 = 7.07 \times 10^6$ m; $g = GM \div r^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \div (7.07 \times 10^6)^2 = 7.97\ N\ kg^{-1}$ (Achieved).

6. Calculate the satellite's orbital speed.
   Gravity supplies the centripetal force, so $v^2 = GM \div r$, or $v = \sqrt{(gr)} = \sqrt{(7.966 \times 7.07 \times 10^6)} = 7.50 \times 10^3\ m\ s^{-1}$ (Merit).

7. Calculate its orbital period in minutes.
   $T = 2\pi r \div v = 2\pi \times 7.07 \times 10^6 \div 7505 = 5919$ s, which is 98.7 minutes (Excellence for the full chain with the radius from Earth's centre).

8. Explain why the satellite keeps a constant speed without using its engines, even though a force acts on it the whole time.
   The only force is gravity, pointing to the Earth's centre, which is at right angles to the velocity at every point of a circular orbit. A force at right angles to the motion does no work, so the kinetic energy and speed do not change; the force only changes the direction, which gives the centripetal acceleration that keeps the satellite on the circle. With no air resistance at this height, nothing removes energy, so no engine is needed (Excellence links the force direction, work and the circular path).

9. Correct this statement: "The free-body diagram shows weight, tension and a centripetal force."
   Centripetal force is the resultant of the real forces, so draw only weight and tension; their sum points to the centre.

10. Correct this statement: "For a satellite 700 km up, $F = GMm \div (7.00 \times 10^5)^2$."
    r is measured from Earth's centre: $r = 6.37 \times 10^6 + 7.00 \times 10^5 = 7.07 \times 10^6$ m.

11. Correct this statement: "Astronauts in orbit are weightless because there is no gravity up there."
    At the height of the International Space Station, about 400 km up, gravity is still about 90% of its surface value; they are in free fall with their craft, so nothing pushes on them.

12. Correct this statement: "A heavier satellite must go faster to stay in the same orbit."
    The satellite's mass cancels in $GMm \div r^2 = mv^2 \div r$, so every satellite at radius r has the same speed.