Year 13 Science: Centre of mass and momentum in two dimensions

Name: _____ Date: _____

Try these without your notes first, then check the answer key.

Practice

1. A 4.0 kg mass sits at x = 0 and a 1.0 kg mass at x = 2.5 m. Where is the centre of mass?

2. A 0.0580 kg tennis ball hits a racquet at 20.0 m s^{-1} and returns straight back at 30.0 m s^{-1}. Contact lasts 4.00 ms. Find the average force on the ball.

3. A 0.400 kg ball moving east at 5.00 m s^{-1} hits a 0.600 kg ball moving north at 2.50 m s^{-1}, and they stick together. Find their velocity.

4. A 3.00 kg object at rest explodes into three 1.00 kg pieces. One goes east at 6.00 m s^{-1} and one north at 8.00 m s^{-1}. Find the velocity of the third piece.

Exam-style question: a tackle

In a rugby match, Sione (95.0 kg) runs east at 6.00 m s^{-1}. Nikau (110 kg) runs at 5.00 m s^{-1} in a direction 60.0° north of west. Nikau tackles Sione and they move off together. Ignore friction from the ground during the tackle.

5. Calculate the size of each player's momentum before the tackle.

6. Calculate the velocity of the two players just after the tackle. A vector diagram may help.

7. The tackle lasts 0.300 s. Calculate the size of the average force on Sione.

8. Explain why momentum is conserved in the tackle but kinetic energy is not.

Fix the mistake

9. Correct this statement: "Total momentum = 18 000 + 21 000 = 39 000 kg m s^{-1} for the two cars."

10. Correct this statement: "The ball hits and leaves at 12.0 m s^{-1}, so its momentum did not change and there was no impulse."

11. Correct this statement: "Momentum is conserved, so kinetic energy must be conserved as well."

12. Correct this statement: "The angle of 49.4° is just given as the answer, with no reference direction."

Answer key (for parents and tutors)

1. A 4.0 kg mass sits at x = 0 and a 1.0 kg mass at x = 2.5 m. Where is the centre of mass?
   $x_{cm} = (4.0 \times 0 + 1.0 \times 2.5) \div 5.0 = 0.50$ m from the 4.0 kg mass.

2. A 0.0580 kg tennis ball hits a racquet at 20.0 m s^{-1} and returns straight back at 30.0 m s^{-1}. Contact lasts 4.00 ms. Find the average force on the ball.
   Taking the return direction as positive, $\Delta p = 0.0580 \times (30.0 - (-20.0)) = 0.0580 \times 50.0 = 2.90$ kg m s^{-1}; $F = 2.90 \div 0.00400 = 725$ N, in the direction the ball returns.

3. A 0.400 kg ball moving east at 5.00 m s^{-1} hits a 0.600 kg ball moving north at 2.50 m s^{-1}, and they stick together. Find their velocity.
   p east $= 0.400 \times 5.00 = 2.00$, p north $= 0.600 \times 2.50 = 1.50$ kg m s^{-1}; $p = \sqrt{(2.00^2 + 1.50^2)} = 2.50$ kg m s^{-1}; $v = 2.50 \div 1.00 = 2.50$ m s^{-1}; $\tan\theta = 1.50 \div 2.00$, $\theta = 36.9°$ north of east.

4. A 3.00 kg object at rest explodes into three 1.00 kg pieces. One goes east at 6.00 m s^{-1} and one north at 8.00 m s^{-1}. Find the velocity of the third piece.
   Total momentum stays zero, so the third piece has momentum equal and opposite to the sum of the other two: $\sqrt{(6.00^2 + 8.00^2)} = 10.0$ kg m s^{-1}, so $v = 10.0$ m s^{-1}; the other two add to a direction 53.1° north of east ($\tan\theta = 8.00 \div 6.00$), so the third piece moves at 10.0 m s^{-1} at 53.1° south of west.

5. Calculate the size of each player's momentum before the tackle.
   Sione: $95.0 \times 6.00 = 570$ kg m s^{-1}; Nikau: $110 \times 5.00 = 550$ kg m s^{-1} (Achieved for both).

6. Calculate the velocity of the two players just after the tackle. A vector diagram may help.
   Components with east and north positive: Sione (570, 0); Nikau ($-550\cos 60.0°$, $550\sin 60.0°$) = (−275, 476.3). Total (295, 476.3) kg m s^{-1}, size $\sqrt{(295^2 + 476.3^2)} = 560.3$ kg m s^{-1}; $v = 560.3 \div 205 = 2.73$ m s^{-1}; $\tan\theta = 476.3 \div 295$, $\theta = 58.2°$ north of east (Merit for the method with one slip, Excellence for the size and direction).

7. The tackle lasts 0.300 s. Calculate the size of the average force on Sione.
   Sione's velocity after is (1.439, 2.323) m s^{-1}, so $\Delta v = (1.439 - 6.00, 2.323) = (-4.561, 2.323)$; $\Delta p = 95.0 \times \sqrt{(4.561^2 + 2.323^2)} = 486.3$ kg m s^{-1}; $F = 486.3 \div 0.300 = 1620$ N (Excellence; Nikau's change in momentum has the same size, the other way).

8. Explain why momentum is conserved in the tackle but kinetic energy is not.
   During the tackle the forces between the players are an internal action and reaction pair, equal in size, opposite in direction and acting for the same time, so the impulses cancel and the total momentum is unchanged. Kinetic energy falls from 3085 J to $\frac{1}{2} \times 205 \times 2.73^2 = 766$ J because the players deform and grip, so work is done converting kinetic energy into heat and sound; this is an inelastic collision (Excellence links the internal forces to momentum and the energy change to the inelastic collision).

9. Correct this statement: "Total momentum = 18 000 + 21 000 = 39 000 kg m s^{-1} for the two cars."
   Momenta at right angles add as vectors: $\sqrt{(18\,000^2 + 21\,000^2)} = 27\,700$ kg m s^{-1}.

10. Correct this statement: "The ball hits and leaves at 12.0 m s^{-1}, so its momentum did not change and there was no impulse."
    The speed is the same but the direction changed, so $\Delta p = p_{final} - p_{initial}$ is not zero.

11. Correct this statement: "Momentum is conserved, so kinetic energy must be conserved as well."
    Kinetic energy is conserved only in elastic collisions; momentum is conserved in every collision without an external force.

12. Correct this statement: "The angle of 49.4° is just given as the answer, with no reference direction."
    Give a direction against something fixed, such as 49.4° north of east or measured from car A's original path.