Year 13 Science: DC circuits, internal resistance and Kirchhoff's laws

Name: _____ Date: _____

Try these without your notes first, then check the answer key.

Practice

1. An AA cell of EMF 1.50 V and internal resistance 0.400 Ω is connected to a 2.60 Ω resistor. Find the current and the terminal voltage.

2. A battery reads 9.00 V on open circuit and 8.40 V when it supplies 0.300 A. Find its internal resistance.

3. Currents of 2.0 A and 0.5 A flow into a junction and 1.2 A flows out along one wire. What current flows in the only other wire, and which way?

4. An e-bike battery has an EMF of 36.0 V and internal resistance 0.150 Ω. Climbing a hill, the motor draws 12.0 A. Find the terminal voltage and the power wasted in the battery.

Exam-style question: two boat batteries

On a fishing boat in the Hauraki Gulf, battery A (EMF 12.0 V, internal resistance 0.200 Ω) runs a bilge pump modelled as a 4.00 Ω resistor.

5. Calculate the current in the pump and the terminal voltage of battery A.

6. Explain why the terminal voltage is less than the EMF.

7. An older battery B (EMF 11.4 V, internal resistance 0.300 Ω) is connected in parallel with A, positive to positive. Use Kirchhoff's laws to calculate the current through each battery.

8. Explain what the sign of battery B's current tells you, and why connecting it this way wastes energy.

Fix the mistake

9. Correct this statement: "$I = \varepsilon \div R = 9.00 \div 3.50 = 2.57$ A, leaving out the battery."

10. Correct this statement: "The EMF of the battery drops when a bigger current is drawn."

11. Correct this statement: "The answer for I_2 came out negative, so I must have made a mistake."

12. Correct this statement: "In the loop equation every EMF and every IR term is added as positive."

Answer key (for parents and tutors)

1. An AA cell of EMF 1.50 V and internal resistance 0.400 Ω is connected to a 2.60 Ω resistor. Find the current and the terminal voltage.
   $I = \varepsilon \div (R + r) = 1.50 \div 3.00 = 0.500$ A; $V = IR = 0.500 \times 2.60 = 1.30$ V (or $1.50 - 0.500 \times 0.400 = 1.30$ V).

2. A battery reads 9.00 V on open circuit and 8.40 V when it supplies 0.300 A. Find its internal resistance.
   Lost volts = 9.00 − 8.40 = 0.60 V, so $r = 0.60 \div 0.300 = 2.00\ \Omega$ (2.0 Ω).

3. Currents of 2.0 A and 0.5 A flow into a junction and 1.2 A flows out along one wire. What current flows in the only other wire, and which way?
   In = 2.5 A, so out = 2.5 A: the other wire carries $2.5 - 1.2 = 1.3$ A out of the junction.

4. An e-bike battery has an EMF of 36.0 V and internal resistance 0.150 Ω. Climbing a hill, the motor draws 12.0 A. Find the terminal voltage and the power wasted in the battery.
   $V = 36.0 - 12.0 \times 0.150 = 34.2$ V; $P = I^2r = 12.0^2 \times 0.150 = 21.6$ W.

5. Calculate the current in the pump and the terminal voltage of battery A.
   $I = 12.0 \div (4.00 + 0.200) = 2.857$ A, about 2.86 A; $V = 12.0 - 2.857 \times 0.200 = 11.4$ V (Achieved).

6. Explain why the terminal voltage is less than the EMF.
   Some of the energy each coulomb gains in the battery is turned into heat as it passes through the internal resistance; these lost volts, $Ir = 0.571$ V, are not available at the terminals, so $V = \varepsilon - Ir$ is less than ε (Merit).

7. An older battery B (EMF 11.4 V, internal resistance 0.300 Ω) is connected in parallel with A, positive to positive. Use Kirchhoff's laws to calculate the current through each battery.
   Let the terminal voltage be V and the currents out of the positive terminals be I_A and I_B. Loop through A: $12.0 - 0.200I_A = V$. Loop through B: $11.4 - 0.300I_B = V$. Junction: $I_A + I_B = V \div 4.00$. Substituting: $(12.0 - V) \div 0.200 + (11.4 - V) \div 0.300 = V \div 4.00$, so $98.0 = 8.5833V$ and $V = 11.4175$ V (11.42 V to 4 significant figures). Keeping the unrounded V, $I_A = (12.0 - 11.4175) \div 0.200 = 2.91$ A and $I_B = (11.4 - 11.4175) \div 0.300 = -0.0583$ A, with 2.85 A in the pump (Excellence for all three).

8. Explain what the sign of battery B's current tells you, and why connecting it this way wastes energy.
   I_B is negative, so the current flows into battery B's positive terminal: battery A is charging battery B as well as running the pump. Because B's EMF is lower than the terminal voltage, A pushes current backwards through it, and both internal resistances turn that extra current into heat, so A's energy is wasted rather than all reaching the pump (Excellence links the sign, the EMFs and the energy).

9. Correct this statement: "$I = \varepsilon \div R = 9.00 \div 3.50 = 2.57$ A, leaving out the battery."
   The internal resistance is in series with the circuit: $I = \varepsilon \div (R + r) = 9.00 \div 4.00 = 2.25$ A.

10. Correct this statement: "The EMF of the battery drops when a bigger current is drawn."
    The EMF stays the same; the terminal voltage $\varepsilon - Ir$ drops because the lost volts grow.

11. Correct this statement: "The answer for I_2 came out negative, so I must have made a mistake."
    A negative current means the real current flows opposite to the arrow you drew; keep the sign in the other equations.

12. Correct this statement: "In the loop equation every EMF and every IR term is added as positive."
    Go round in one direction: an EMF is negative if you cross it from + to −, and an IR term is negative if you go against the current.