Year 8 · Probability: Testing Your Luck: Experimental Probability

Name: _____ Date: _____

Hi! This sheet is all about doing chance experiments and working out probabilities from your results. You'll see how experimental probability compares to what should happen in theory, and why doing MORE trials usually gets you closer to the truth.

Warm-up

1. Ava flips a coin 40 times and gets 22 heads. What is the experimental probability of getting heads? Give your answer as a fraction and a decimal.

2. A spinner is spun 50 times and lands on red 15 times. What is the experimental probability of landing on red?

3. A die is rolled 60 times and a six comes up 8 times. What is the experimental probability of rolling a six? Give a fraction and a rounded decimal.

4. What is the theoretical probability of rolling a 4 on a fair six-sided die?

Practice

5. Sione rolls a fair die 30 times and records: 1 → 4 times, 2 → 6 times, 3 → 3 times, 4 → 7 times, 5 → 5 times, 6 → 5 times. Calculate the experimental probability of each outcome.

6. Using Sione's results above, compare his experimental probability of rolling a 4 to the theoretical probability. Why are they different?

7. At the Avondale School gala, Mereana runs a stall with a bag holding coloured cubes. She pulls one out, records the colour, and puts it back, doing this 100 times. Results: green 46, yellow 34, purple 20. Find the experimental probability of each colour.

8. Mereana's bag actually contains 10 cubes: 5 green, 3 yellow, 2 purple. Work out the theoretical probability of each colour and compare it with her experimental results.

9. Two students both roll the same fair die. Tama rolls it 20 times and gets 5 sixes (P = 0.25). Kiri rolls it 120 times and gets 22 sixes (P ≈ 0.18). Whose experimental probability is a better estimate of the true value of 1/6 ≈ 0.17, and why?

10. Explain, in your own words, what the Law of Large Numbers tells us about a chance experiment.

Challenge

11. Ropata suspects one of his dice is weighted. He rolls it 200 times and gets a six 60 times. (a) Find the experimental probability of a six. (b) What would you expect the count of sixes to be for a fair die over 200 rolls? (c) Do you think the die is weighted? Explain using the Law of Large Numbers.

12. A spinner has 4 unequal sections: A, B, C, D. In 50 spins the results are A 8, B 22, C 14, D 6. (a) Find the experimental probability of each section. (b) The spinner is then spun 500 times, giving A 60, B 250, C 130, D 60. Find the new experimental probabilities. (c) The true (theoretical) probabilities are A 0.12, B 0.50, C 0.26, D 0.12. Which set of trials gave estimates closer to the true values, and what does this show?

Answer key (for parents and tutors)

1. Ava flips a coin 40 times and gets 22 heads. What is the experimental probability of getting heads? Give your answer as a fraction and a decimal.
   Experimental probability = (times it happened) ÷ (total trials) = 22 ÷ 40 = 22/40 = 11/20 = 0.55.
2. A spinner is spun 50 times and lands on red 15 times. What is the experimental probability of landing on red?
   15 ÷ 50 = 15/50 = 3/10 = 0.30.
3. A die is rolled 60 times and a six comes up 8 times. What is the experimental probability of rolling a six? Give a fraction and a rounded decimal.
   8 ÷ 60 = 8/60 = 2/15 ≈ 0.13 (rounded to 2 decimal places).
4. What is the theoretical probability of rolling a 4 on a fair six-sided die?
   A fair die has 6 equally likely outcomes, and only one is a 4, so the theoretical probability = 1/6 ≈ 0.17.
5. Sione rolls a fair die 30 times and records: 1 → 4 times, 2 → 6 times, 3 → 3 times, 4 → 7 times, 5 → 5 times, 6 → 5 times. Calculate the experimental probability of each outcome.
   Total trials = 4+6+3+7+5+5 = 30 (correct). Divide each count by 30: P(1)=4/30=2/15≈0.13; P(2)=6/30=1/5=0.20; P(3)=3/30=1/10=0.10; P(4)=7/30≈0.23; P(5)=5/30=1/6≈0.17; P(6)=5/30=1/6≈0.17. (The probabilities add to 1.)
6. Using Sione's results above, compare his experimental probability of rolling a 4 to the theoretical probability. Why are they different?
   Experimental P(4) = 7/30 ≈ 0.23. Theoretical P(4) = 1/6 ≈ 0.17. They differ because 30 trials is a small number, so random chance means some outcomes come up more often than expected just by luck. Doing many more rolls would usually bring the experimental value closer to 0.17.
7. At the Avondale School gala, Mereana runs a stall with a bag holding coloured cubes. She pulls one out, records the colour, and puts it back, doing this 100 times. Results: green 46, yellow 34, purple 20. Find the experimental probability of each colour.
   Total = 46+34+20 = 100. P(green)=46/100=0.46; P(yellow)=34/100=0.34; P(purple)=20/100=0.20. (These add to 1.00.)
8. Mereana's bag actually contains 10 cubes: 5 green, 3 yellow, 2 purple. Work out the theoretical probability of each colour and compare it with her experimental results.
   Theoretical: P(green)=5/10=0.50; P(yellow)=3/10=0.30; P(purple)=2/10=0.20. Compared with experiment (0.46, 0.34, 0.20): purple matches exactly, while green and yellow are a little off due to random variation. The values are close because she did 100 trials, a large number of trials.
9. Two students both roll the same fair die. Tama rolls it 20 times and gets 5 sixes (P = 0.25). Kiri rolls it 120 times and gets 22 sixes (P ≈ 0.18). Whose experimental probability is a better estimate of the true value of 1/6 ≈ 0.17, and why?
   Kiri's estimate (≈0.18) is better because it is closer to 0.17 and, more importantly, it is based on 120 trials. According to the Law of Large Numbers, more trials give an experimental probability closer to the theoretical probability. Tama's 20 trials are easily affected by luck.
10. Explain, in your own words, what the Law of Large Numbers tells us about a chance experiment.
    Sample answer: The Law of Large Numbers says that as you do more and more trials of the same experiment, the experimental probability of each outcome gets closer and closer to its theoretical probability. A small number of trials can be far off due to random luck, but a large number of trials usually 'settles down' near the true value.
11. Ropata suspects one of his dice is weighted. He rolls it 200 times and gets a six 60 times. (a) Find the experimental probability of a six. (b) What would you expect the count of sixes to be for a fair die over 200 rolls? (c) Do you think the die is weighted? Explain using the Law of Large Numbers.
    (a) P(six) = 60/200 = 0.30. (b) For a fair die, expected sixes = 200 × 1/6 ≈ 33 (about 33 sixes). (c) The experimental probability 0.30 is much higher than the theoretical 0.17, and 60 sixes is nearly double the expected 33. Because 200 is a large number of trials, the Law of Large Numbers says the result should be close to 1/6 if the die were fair. Since it is so far off, it is strong evidence the die is weighted towards six.

12. A spinner has 4 unequal sections: A, B, C, D. In 50 spins the results are A 8, B 22, C 14, D 6. (a) Find the experimental probability of each section. (b) The spinner is then spun 500 times, giving A 60, B 250, C 130, D 60. Find the new experimental probabilities. (c) The true (theoretical) probabilities are A 0.12, B 0.50, C 0.26, D 0.12. Which set of trials gave estimates closer to the true values, and what does this show?
(a) 50 spins: P(A)=8/50=0.16; P(B)=22/50=0.44; P(C)=14/50=0.28; P(D)=6/50=0.12. (b) 500 spins: P(A)=60/500=0.12; P(B)=250/500=0.50; P(C)=130/500=0.26; P(D)=60/500=0.12. (c) The 500-spin results match the true values almost exactly (0.12, 0.50, 0.26, 0.12), while the 50-spin results were more spread out (especially A and B). This demonstrates the Law of Large Numbers: increasing the number of trials from 50 to 500 made the experimental probabilities settle very close to the theoretical probabilities.