Year 9 · Probability: Testing Theory Against Trials

Name: _____ Date: _____

Hi! This sheet is all about the difference between what we expect to happen (theoretical probability) and what actually happens when we run experiments (experimental probability). You'll list outcomes, crunch some numbers, and see the Law of Large Numbers in action. Grab a calculator and let's go!

Warm-up

1. A fair six-sided dice is rolled once. What is the theoretical probability of rolling a 4? Give your answer as a fraction and a percentage (to the nearest whole percent).

2. Aria tossed a coin 30 times and got 18 heads. What is the experimental probability of getting a head? Give your answer as a fraction in simplest form.

3. A bag holds 5 red, 3 blue and 2 green marbles. One marble is drawn at random. What is the theoretical probability it is blue?

4. Are the events 'rolling an even number' and 'rolling a 3' on a single dice roll mutually exclusive? Explain in one sentence.

Practice

5. A spinner is divided into 4 equal sections coloured red, yellow, blue and white. List the full sample space for one spin, then state the theoretical probability of landing on yellow.

6. Two fair coins are tossed together. Use a systematic list or table to write every possible outcome, then find the probability of getting exactly one head.

7. Tane rolled a fair dice 60 times. He recorded: 1 appeared 8 times, 2 appeared 11 times, 3 appeared 9 times, 4 appeared 12 times, 5 appeared 10 times, 6 appeared 10 times. Find the experimental probability of rolling a 4, and compare it with the theoretical probability.

8. At the Avondale Sunday Market, a lucky-dip drum contains 40 tickets: 4 are winners and 36 are blanks. Mere pays $2 for one ticket. What is the theoretical probability she draws a winning ticket? Give your answer as a fraction in simplest form and as a percentage.

9. A class ran a simulation of tossing a drawing pin 100 times to see how often it lands 'point up'. It landed point up 62 times. (a) Why can this probability only be found by experiment, not by theory? (b) Estimate the probability the pin lands 'point down'.

10. Ben used a computer to simulate rolling a fair dice. After 30 rolls the experimental probability of a 6 was 0.10. After 1000 rolls it was 0.165. Explain, using the Law of Large Numbers, why the second result is closer to the theoretical probability.

Challenge

11. A fair coin is tossed and a fair dice is rolled. (a) Draw or list all outcomes using a systematic method. (b) Find the probability of getting a head AND a number greater than 4. (c) Explain why you cannot simply add P(head) and P(number greater than 4) to get this answer.

12. A bag has 3 kiwifruit stickers and 2 feijoa stickers. Two stickers are drawn one after the other WITHOUT replacing the first. (a) Are the two draws independent or dependent? (b) Use a tree diagram (describe the branches) to find the probability that both stickers are kiwifruit.

Answer key (for parents and tutors)

1. A fair six-sided dice is rolled once. What is the theoretical probability of rolling a 4? Give your answer as a fraction and a percentage (to the nearest whole percent).
   There is 1 favourable outcome (a 4) out of 6 equally likely outcomes. Probability = 1/6. As a percentage: 1 ÷ 6 = 0.1666… ≈ 17%.

2. Aria tossed a coin 30 times and got 18 heads. What is the experimental probability of getting a head? Give your answer as a fraction in simplest form.
   Experimental probability = number of heads ÷ total trials = 18/30. Simplify by dividing both by 6: 18 ÷ 6 = 3, 30 ÷ 6 = 5. So the answer is 3/5.

3. A bag holds 5 red, 3 blue and 2 green marbles. One marble is drawn at random. What is the theoretical probability it is blue?
   Total marbles = 5 + 3 + 2 = 10. Blue marbles = 3. Probability = 3/10 (or 0.3, or 30%).

4. Are the events 'rolling an even number' and 'rolling a 3' on a single dice roll mutually exclusive? Explain in one sentence.
   Yes, they are mutually exclusive because a single roll cannot be both even and equal to 3 at the same time, 3 is an odd number, so the two events share no common outcome.

5. A spinner is divided into 4 equal sections coloured red, yellow, blue and white. List the full sample space for one spin, then state the theoretical probability of landing on yellow.
   Sample space (all possible outcomes) = {red, yellow, blue, white}. There are 4 equally likely outcomes. Yellow is 1 of them, so P(yellow) = 1/4 = 0.25 = 25%.

6. Two fair coins are tossed together. Use a systematic list or table to write every possible outcome, then find the probability of getting exactly one head.
   Using H for heads and T for tails, the sample space is: HH, HT, TH, TT, that is 4 equally likely outcomes. 'Exactly one head' happens for HT and TH, which is 2 outcomes. P(exactly one head) = 2/4 = 1/2 = 50%.

7. Tane rolled a fair dice 60 times. He recorded: 1 appeared 8 times, 2 appeared 11 times, 3 appeared 9 times, 4 appeared 12 times, 5 appeared 10 times, 6 appeared 10 times. Find the experimental probability of rolling a 4, and compare it with the theoretical probability.
   Experimental probability of a 4 = 12/60 = 1/5 = 0.2 = 20%. Theoretical probability = 1/6 ≈ 0.167 ≈ 17%. The experimental value (20%) is a little higher than the theoretical value (17%). This is normal variation from a limited number of trials; with many more rolls we would expect the experimental value to move closer to 1/6.

8. At the Avondale Sunday Market, a lucky-dip drum contains 40 tickets: 4 are winners and 36 are blanks. Mere pays $2 for one ticket. What is the theoretical probability she draws a winning ticket? Give your answer as a fraction in simplest form and as a percentage.
   Winners = 4, total tickets = 40. Probability = 4/40. Simplify by dividing both by 4: 4 ÷ 4 = 1, 40 ÷ 4 = 10, giving 1/10. As a percentage: 1 ÷ 10 = 0.1 = 10%.

9. A class ran a simulation of tossing a drawing pin 100 times to see how often it lands 'point up'. It landed point up 62 times. (a) Why can this probability only be found by experiment, not by theory? (b) Estimate the probability the pin lands 'point down'.
   (a) A drawing pin is not a regular symmetrical shape, so its two outcomes are not equally likely. There is no way to calculate the probability by reasoning about equal outcomes, so we must run an experiment and use relative frequency. (b) Point up happened 62 times, so point down happened 100 − 62 = 38 times. Estimated P(point down) = 38/100 = 0.38 = 38%.

10. Ben used a computer to simulate rolling a fair dice. After 30 rolls the experimental probability of a 6 was 0.10. After 1000 rolls it was 0.165. Explain, using the Law of Large Numbers, why the second result is closer to the theoretical probability.
    The theoretical probability of a 6 is 1/6 ≈ 0.167. With only 30 rolls, a few unlucky or lucky results have a big effect, so 0.10 is quite far from 0.167. With 1000 rolls, individual variations average out, so the experimental probability (0.165) sits much closer to 0.167. This is the Law of Large Numbers: as the number of trials increases, the experimental probability approaches the theoretical probability.

11. A fair coin is tossed and a fair dice is rolled. (a) Draw or list all outcomes using a systematic method. (b) Find the probability of getting a head AND a number greater than 4. (c) Explain why you cannot simply add P(head) and P(number greater than 4) to get this answer.
(a) The sample space pairs each coin result with each dice result. Listing them: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6, that is 12 equally likely outcomes (2 × 6 = 12). (b) 'Head AND greater than 4' means a head with a 5 or 6: that is H5 and H6, so 2 outcomes. P = 2/12 = 1/6 ≈ 0.167. (c) The coin and dice are independent events happening together (a joint event). Adding P(head) = 1/2 and P(>4) = 2/6 = 1/3 would give 5/6, which counts full separate events rather than the single combined outcome. For independent joint events we multiply: 1/2 × 1/3 = 1/6, which matches the count.

12. A bag has 3 kiwifruit stickers and 2 feijoa stickers. Two stickers are drawn one after the other WITHOUT replacing the first. (a) Are the two draws independent or dependent? (b) Use a tree diagram (describe the branches) to find the probability that both stickers are kiwifruit.
(a) They are dependent, because removing the first sticker changes what is left in the bag, so the second draw's probabilities depend on the first outcome. (b) First draw: P(kiwifruit) = 3/5. If a kiwifruit was taken, 2 kiwifruit remain out of 4 stickers, so the second draw P(kiwifruit) = 2/4 = 1/2. Multiplying along the branches: P(both kiwifruit) = 3/5 × 1/2 = 3/10 = 0.3 = 30%.